Introductory Algebra
Chapter 2 Review

Objective [2.1a] Determine whether a given number is a solution of a given equation.

Brief Procedure	Example	Practice Exercise
Substitute the given number in the equation and determine if a true equation results.	Determine whether -3 is a solution of $x+4=1$. $x+4=1$ $-3+4 \; ? \; 1$ $1 \;\vert\;$ TRUE Since the left-hand and right-hand sides are the same, we have a true equation so -3 is a solution.	1. Determine whether 5 is a solution of $9x=42$. A. Yes B. No

Objective [2.1b] Solve equations using the addition principle.

Brief Procedure	Example	Practice Exercise
For any real numbers a, b, and c, $a=b$ is equivalent to $a+c=b+c$. Add the same number on both sides of the equation to get the variable alone. Since $a+(-c)=b+(-c)$ is equivalent to $a-c=b-c$, we can also subtract the same number on both sides of the equation.	Solve: $x+4=9$. We subtract 4 on both sides of the equation to get x alone. $x+4=9$ $x+4-4=9-4$ $x+0=5$ $x=5$ The solution is 5.	2. Solve: $y-3=-1$. A. -4 B. -2 C. 2 D. 4

Objective [2.2a] Solve equations using the multiplication principle.

Brief Procedure	Example	Practice Exercise
For any real numbers a, b, and c, $c \neq 0$, $a=b$ is equivalent to $a \cdot c = b \cdot c$. Multiply by the same number on both sides of the equation to get the variable alone. For $c \neq 0$, $a \cdot \frac{1}{c} = b \cdot \frac{1}{c}$ is equivalent to $\frac{a}{c} = \frac{b}{c}$, so we can also divide by the same number on both sides of the equation.	Solve: $54=-9y$. We divide by -9 on both sides of the equation to get y alone. $54=-9y$ $\frac{54}{-9} = \frac{-9y}{-9}$ $-6 = 1 \cdot y$ $-6 = y$ The solution is -6.	3. Solve: $6x=-42$. A. 7 B. -7 C. -36 D. -48

Objective [2.3a] Solve equations using both the addition and the multiplication principles.		
Brief Procedure	Example	Practice Exercise
First use the addition principle to isolate the term that contains the variable. Then use the multiplication principle to get the variable by itself.	Solve: $2x-5=3$. $$\begin{aligned}2x-5&=3\\2x-5+5&=3+5\\2x&=8\\\frac{2x}{2}&=\frac{8}{2}\\x&=4\end{aligned}$$ The solution is 4.	4. Solve: $3y+1=-8$. A. $-\frac{7}{3}$ B. $-\frac{8}{3}$ C. -3 D. -11
Objective [2.3b] Solve equations in which like terms may need to be collected.		
Brief Procedure	Example	Practice Exercise
If there are like terms on one side of the equation, collect them before using the addition or multiplication principle. If there are like terms on opposite sides of the equation, use the addition principle to get all terms with a variable on one side and all numbers on the other.	Solve: $2y-1=-3y-8+2$. $$\begin{aligned}2y-1&=-3y-8+2\\2y-1&=-3y-6\\2y-1+1&=-3y-6+1\\2y&=-3y-5\\2y+3y&=-3y-5+3y\\5y&=-5\\\frac{5y}{5}&=\frac{-5}{5}\\y&=-1\end{aligned}$$ The solution is -1.	5. Solve: $3x-5-x=6x+7$. A. -3 B. -1 C. $\frac{1}{4}$ D. $\frac{3}{2}$
Objective [2.3c] Solve equations by first removing parentheses and collecting like terms.		
Brief Procedure	Example	Practice Exercise
If an equation contains parentheses, first use the distributive laws to remove them. Then collect like terms, if necessary, and use the addition and multiplication principles to complete the solution of the equation.	Solve: $8b-2(3b+1)=10$. $$\begin{aligned}8b-2(3b+1)&=10\\8b-6b-2&=10\\2b-2&=10\\2b-2+2&=10+2\\2b&=12\\\frac{2b}{2}&=\frac{12}{2}\\b&=6\end{aligned}$$ The solution is 6.	6. Solve: $3(n-4)=2(n+1)$. A. -5 B. -2 C. 5 D. 14

Objective [2.4a] Solve applied problems by translating to equations.		
Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process. 1. *Familiarize* yourself with the problem situation. 2. *Translate* the problem to an equation. 3. *Solve* the equation. 4. *Check* the answer in the original problem. 5. *State* the answer to the problem clearly.	A 12-ft pipe is cut into three pieces. The second piece is three times as long as the first. The third piece is twice as long as the first. How long is each piece? 1. *Familiarize.* Let $x =$ the length of the first piece of pipe. Then $3x =$ the length of the second piece and $2x =$ the length of the third piece. 2. *Translate.* We use the fact that the sum of the lengths is 12 feet. $\underbrace{\text{Length of first piece}}_{x}\ \underset{+}{\text{plus}}\ \underbrace{\text{length of second piece}}_{3x}\ \underset{+}{\text{plus}}\ \underbrace{\text{length of third piece}}_{2x}\ \underset{=}{\text{is}}\ \underbrace{\text{total length}}_{12}$ 3. *Solve.* We solve the equation. $x + 3x + 2x = 12$ $6x = 12$ $\frac{6x}{6} = \frac{12}{6}$ $x = 2$ If $x = 2$, then $3x = 3 \cdot 2$, or 6, and $2x = 2 \cdot 2$, or 4. 4. *Check.* The second piece, 6 ft, is three times as long as the first, 2 ft, and the third piece, 4 ft, is twice as long as the first. Also, the lengths total 2 ft + 6 ft + 4 ft, or 12 ft. The answer checks. 5. *State.* The first piece of pipe is 2 ft long, the second piece is 6 ft, and the third piece is 4 ft.	7. The perimeter of a rectangular rug is 40 ft. The width is 4 ft less than the length. Find the dimensions of the rug. A. The width is 8 ft. B. The width is 10 ft. C. The width is 12 ft. D. The width is 16 ft.

Objective [2.5a] Solve applied problems involving percent.		
Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	An investment is made at 8% simple interest for 1 year. It grows to \$2700. How much was originally invested? 1. *Familiarize.* Recall the formula for simple interest, $I = Prt$, where I is the interest, P is the principal (the amount invested), r is the interest rate, and t is the length of time the principal is invested. Let $x =$ the amount originally invested. Then the interest earned in 1 year is $x \cdot 8\% \cdot 1$, or $8\%x$. The principal plus the interest is the amount to which the investment grows after 1 year. 2. *Translate.* We reword the problem and translate. Principal plus Interest = Amount $x + 8\%x = 2700$ 3. *Solve.* We solve the equation. $x + 8\%x = 2700$ $x + 0.08x = 2700$ $1x + 0.08x = 2700$ $1.08x = 2700$ $\frac{1.08x}{1.08} = \frac{2700}{1.08}$ $x = 2500$ 4. *Check.* We check by finding 8% of \$2500 and adding it to \$2500: $8\% \times \$2500 = 0.08 \times \$2500 = \$200$ and $\$2500 + \$200 = \$2700$. The answer checks. 5. *State.* The original investment was \$2500.	8. After a 25% price reduction, a CD is on sale for \$13.50. What was the original price? A. \$16 B. \$16.88 C. \$18 D. \$19.75

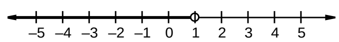
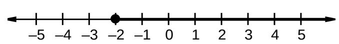
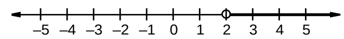
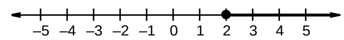
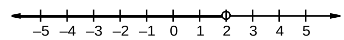
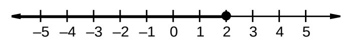
Objective [2.6a] Evaluate formulas and solve a formula for a specified letter.

Brief Procedure	Example	Practice Exercises
To evaluate a formula for a given value of the variable, substitute the value for the variable and carry out the resulting calculations.	The formula $d = 65t$ gives the distance d that is traveled in t hours at a speed of 65 mph. Suppose you travel at 65 mph for 4 hours. How far have you traveled? We substitute 4 for t and carry out the calculation. $d = 65 \cdot 4 = 260$ You have traveled 260 miles.	9. Using the formula $d = 65t$, find the distance traveled at 65 mph for 3 hours. A. 185 miles B. 195 miles C. 205 miles D. 225 miles
To solve a formula for a specified letter, identify the letter and: 1. Multiply on both sides to clear fractions or decimals, if that is needed. 2. Collect like terms on each side, if necessary. 3. Get all terms with the letter to be solved for on one side of the equation and all other terms on the other side. 4. Collect like terms again, if necessary. 5. Solve for the letter in question.	Solve for b: $A = \frac{a+b}{2}$. $A = \frac{a+b}{2}$ $2 \cdot A = 2\left(\frac{a+b}{2}\right)$ $2A = a + b$ $2A - a = b$	10. Solve for h: $A = \frac{1}{2}bh$. A. $h = \frac{A}{2b}$ B. $h = \frac{b}{2A}$ C. $h = \frac{2b}{A}$ D. $h = \frac{2A}{b}$

Objective [2.7a] Determine whether a given number is a solution of an inequality.

Brief Procedure	Example	Practice Exercise
Substitute the given number for the variable and determine if a true inequality results.	Determine whether each number is a solution of $y \leq -4$. a) 2 b) -4 a) Since $2 \leq -4$ is false, 2 is not a solution. b) Since $-4 \leq -4$ is true, -4 is a solution.	11. Determine whether -3 is a solution of $x \geq -5$. A. Yes B. No

Objective [2.7b] Graph an inequality on a number line.

Brief Procedure	Example	Practice Exercise
Shade all points on the number line that are solutions of the given inequality.	Graph each inequality. a) $x < 1$ b) $x \geq -2$ a) The solutions of $x < 1$ are all numbers less than 1. We shade all points to the left of 1. Use an open circle at 1 to indicate that 1 is not part of the graph.  b) The solutions of $x \geq -2$ are all numbers greater than -2 and the number -2 as well. We shade all points to the right of -2, and we use a closed circle at -2 to indicate that -2 is part of the graph. 	12. Graph $x > 2$. A.  B.  C.  D. 

Objective [2.7c] Solve inequalities using the addition principle.

Brief Procedure	Example	Practice Exercise
For any real numbers, a, b, and c: $a < b$ is equivalent to $a + c < b + c$; $a > b$ is equivalent to $a + c > b + c$; $a \leq b$ is equivalent to $a + c \leq b + c$; $a \geq b$ is equivalent to $a + c \geq b + c$. In other words, when we add or subtract the same number on both sides of an inequality, the direction of the inequality symbol is not changed.	Solve: $4x - 2 > 3x + 1$. $4x - 2 > 3x + 1$ $4x - 2 + 2 > 3x + 1 + 2$ $4x > 3x + 3$ $4x - 3x > 3x + 3 - 3x$ $x > 3$ The solution set is $\{x \mid x > 3\}$.	13. Solve: $x + 7 \leq 5$. A. $\{x \mid x \leq -12\}$ B. $\{x \mid x \leq -2\}$ C. $\{x \mid x \leq 2\}$ D. $\{x \mid x \leq 12\}$

Objective [2.7d] Solve inequalities using the multiplication principle.		
Brief Procedure	Example	Practice Exercise
For any real numbers, a, b, and any *positive* number c: $a < b$ is equivalent to $ac < bc$; $a > b$ is equivalent to $ac > bc$. For any real numbers, a, b, and any *negative* number c: $a < b$ is equivalent to $ac > bc$; $a > b$ is equivalent to $ac < bc$. Similar statements hold for $\leq$ and $\geq$. In other words, when we multiply or divide by a positive number on both sides of an inequality, the direction of the inequality symbol stays the same. When we multiply or divide by a negative number on both sides of an inequality, the direction of the inequality symbol is reversed.	Solve: $-3y > 15$. $-3y > 15$ $\frac{-3y}{-3} < \frac{15}{-3}$ $y < -5$ Note that the direction of the inequality symbol was reversed when we divided by -3 on both sides. The solution set is $\{y \mid y < -5\}$.	14. Solve: $-4x \leq -12$. A. $\{x \mid x \leq 3\}$ B. $\{x \mid x \leq -3\}$ C. $\{x \mid x \geq 3\}$ D. $\{x \mid x \geq -3\}$

Objective [2.7e] Solve inequalities using the addition and multiplication principles together.		
Brief Procedure	Example	Practice Exercise
First use the addition principle to isolate the term that contains the variable. Then use the multiplication principle to get the variable by itself.	Solve: $6y + 5 \leq 3y - 7$. $6y + 5 \leq 3y - 7$ $6y + 5 - 3y \leq 3y - 7 - 3y$ $3y + 5 \leq -7$ $3y + 5 - 5 \leq -7 - 5$ $3y \leq -12$ $\frac{3y}{3} \leq \frac{-12}{3}$ $y \leq -4$ The solution set is $\{y \mid y \leq -4\}$.	15. Solve: $8y - 7 > 5y + 2$. A. $\{y \mid y < -3\}$ B. $\{y \mid y > -3\}$ C. $\{y \mid y > \frac{9}{13}\}$ D. $\{y \mid y > 3\}$

Objective [2.8a] Translate number sentences to inequalities.		
Brief Procedure	Example	Practice Exercise
Read the sentence carefully and then translate to an inequality.	Sarah works at least 15 hours each week. $h \geq 15$	16. Joe's budget allows \$20 per week, at most, for entertainment. A. $E < 20$ B. $E \leq 20$ C. $E > 20$ D. $E \geq 20$

Objective [2.8b] Solve applied problems using inequalities.

Brief Procedure	Example	Practice Exercise
Use the five-step problem solving process.	The perimeter of a rectangular patio is not to exceed 60 ft. The length is to be twice the width. What widths will meet these conditions? 1. *Familiarize.* Recall that the formula for the perimeter P of a rectangle is $P = 2l + 2w$, where l is the length and w is the width. Since the length is twice the width, we have $l = 2w$ and then $2l + 2w = 2 \cdot 2w + 2w = 4w + 2w = 6w$. 2. *Translate.* We reword the problem and translate. $\underbrace{\text{Perimeter}}_{\substack{\downarrow \\ 6w}} \quad \underbrace{\text{is less than or equal to}}_{\substack{\downarrow \\ \leq}} \quad \underbrace{60 \text{ ft.}}_{\substack{\downarrow \\ 60}}$ 3. *Solve.* We solve the inequality. $6w \leq 60$ $\frac{6w}{6} \leq \frac{60}{6}$ $w \leq 10$ 4. *Check.* We can obtain a partial check by substituting a number less than or equal to 10 for w. For example, for $w = 9$: $6w = 6 \cdot 9 = 54 \leq 60$. The result is probably correct. 5. *State.* Widths less than or equal to 10 ft will meet the given conditions.	17. Kelly's scores on the first three tests in her physics class are 79, 84, and 68. Determine all scores on the fourth test that will yield an average test score of at least 80. A. Scores of 88 or higher B. Scores of 89 or higher C. Scores of 91 or higher D. Scores of 94 or higher