Algebra Connections

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GCF & Factoring Polynomials

Everything we have done up to this point in Beginning Algebra has been to get ready to apply basic procedures to more involved algebraic concepts such as factoring polynomials.

As a quick review, let's start out by reviewing what factoring is. Factoring is a process to determine what we can multiply to get the given quantity. If means to write an as a product – the reverse of multiplication. In this example, we want to rewrite a polynomial as a product.

Examples: Factors of 12

Possible Solutions: 2*6 or 3*4 or 2*2*3 or [1/2(12)] or (-2*-6) or (-2*2*-3) **Note:** There are many more possible ways to factor 12, but these are representative of many of them.

A useful method of factoring numbers is to completely factor them into positive **prime factors.** A prime number is a number whose only positive factors are 1 and itself. For example 2, 3, 5, and 7 are all examples of prime numbers. Examples of numbers that aren't prime include 4, 6, and 12.

If we completely factor a number into positive prime factors, there will only be one solution. This is why prime factorization is useful. That is the reason for factoring things in this way. For our example above with, **the complete factorization of 12 is: (2*2*3).**

Factoring Polynomials

Factoring polynomials is similar, determine all the terms that were multiplied together to get the original polynomial. For many of us, it is probably easiest to factor in steps – start with the first factors we see and continue until we can't factor anymore. When we cannot find any more factors we will say that the polynomial is completely factored.

Example: $x^2-16 = (x+4)(x-4)$. This is completely factored, we cannot find another way that the factors on the right can be further factored (think: write as product).

Example: $x^4-16 = (x^2+4)(x^2-4)$. This is not completely factored, because the second factor on the right can be further factored. Do you see that it is one of our special products, a difference of squares? Breaking that example down further, we see: **$x^4-16 = (x2+4)(x+2)(x-2)$.**

Greatest Common Factor

One way to factor polynomials, probably the easiest one when applicable, is to factor out the greatest common factor. In general this should **ALWAYS** be the first method we consider whenever factoring.

Start by looking at each terms and determine if there is a factor that is in common to all the terms. If there is, factor it out of the polynomial. Let's take a look at some examples.

Example: $8x^4-4x^3-10x^2$

Notice that we can factor out a 2 from every term, we can also factor out an x^2. Some of us will see that we can factor out a $2x^2$ immediately, but we can also factor this in steps if that helps us get started. **Our solution to this problem is: $2x^2(4x2-2x+5)$.**

Example: $x^3y^2+3x^4y+5x^5y^3$

On closer inspection, we can see that each term has a common factor has x^3y. Of course, we might have seen this in steps, that each term has a factor of x^3 and then that each has a factor of y. It doesn't matter, as long as we keep looking for factors. **Our solution is: $x^3y(y+3x+5x^2y^2)$**

Example: $3x^6-6x^2+3x$

Do you see that each term has a common factor (greatest common factor) of 3x? Please remember, when we factor a 3x out of the last term (3x), we are left with +1. Be careful, forgetting this 1 is a common error when factoring! **Our solution is: 3x(x5-2x+1)**

Example: $9x^2(2x+7)-12x(2x+7)$

At first glance, this one looks strange. It is factored like the other examples, however. There is a 3x in each term and there is also a (2x+7) in each term. Both can be factored out of this polynomial. This leaves a 3x in the first term (because $3x*3x=9x^2$) and a -4 in the second (3x*-4 gives us our original -12x). **The solution is: 3x(2x+7)(3x-4)**

Factoring By Grouping

This is another way to factor polynomials that have 4 terms. It will not work with every 4 term polynomial, but when it can be used, it is a simple and direct approach. It is also a great review of important, basic algebraic procedures. Here is a step-by-step procedure for factoring 4 term polynomials by grouping:

1. Group terms into 2 groups of 2 terms.
2. Factor out the greatest common factor from each of these two groups.
3. If we are left with a common binomial factor, factor it out – then we are done!
4. If not, rearrange the terms and try steps 1-3 again.

Example: $3x^2-2x+12x-8$

1. Group terms into 2 groups of 2 terms: $(3x^2-2x) + (12x-8)$
2. Factor out the greatest common factor from each term: $x(3x-2) + 4(3x-2)$
3. Factor out a common binomial factor. **Our solution is: (3x-2)(x+4).**

That's it! We do not need to do step 4 because we had a common binomial term of (3x-2). Once we factor it out, we are done.

Example: x^4+x-2x^3-2

1. Group terms into 2 groups of 2 terms. **BE CAREFUL WITH POLYNOMIALS WITH A "-" SIGN IN FRONT OF THE 3RD TERM!** The process is the same, but notice that we have a common factor in the 3rd and 4th terms of -1 or just "-". If we factor out this "-" and group the first 2 and last 2 terms together, we get: $(x^4+x)-(2x^3+2)$. **THIS IS AN IMPORTANT STEP WHEN WE HAVE A "-" SIGN IN THE THIRD TERM.**
2. Factor out the greatest common factor from each term: $x(x^3+1)-2(x+1)$
3. Factor out a common binomial factor. **Our solution is: $(x^3+1)(x-2)$.**

Example: $x^5-3x^3-2x^2+6$

1. Group terms into 2 groups of 2 terms. Again, **NOTICE THAT WE HAVE A "-" IN FRONT OF THE THIRD TERM.** Also note the "+" in front of the 4th term. We will still factor out the '-', so that we do not lose track of it. This gives us: $(x^5-3x^3) – (2x^2-6)$.
2. Factor out the greatest common factor from each term: $x^3(x^2-3)-2(x^2-3)$
3. Factor out a common binomial factor. **Our solution is: $(x^2-3)(x^3-2)$.**

Example: $5x-10+x^3-x^2$

1. Group terms into 2 groups of 2 terms. Note that in this case, the 3rd term is positive. Our 2 groups are: $(5x-10) + (x^3-x^2)$
2. Factor out the greatest common factor from each term: $5(x-2)+(x^2-1)$
3. Note that there is no common binomial factor. No grouping will lead to a common factor – **We cannot find a solution by factoring by grouping.**

Example: 3xy+2-3x-2y

1. Note that the first 2 terms have no common factors other than one. If we rearrange the terms, however, we can create 2 groupings with common factors: (3xy-3x)+(-2y+2). **NOTICE THAT WE HAVE A "-" IN FRONT OF THE THIRD TERM.** Also note the "+" in front of the 4th term. We will still factor out the '-', so that we do not lose track of it. This gives us: (3xy-3x)-(2y-2)
2. Factor out the greatest common factor from each term: 3x(y-1)-2(y-1)
3. Factor out a common binomial factor. **Our solution is: (y-1)(3x-2)**

Remember, factoring by grouping can be efficient, but it doesn't work all that often. It is an good review of algebraic procedures. **BE CAREFUL** when there is a "-" in front of the third term. **ALWAYS** factor that out of the third and fourth terms when grouping.

Online Resources

GCF & Factoring by Grouping
http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut27_gcf.htm

Interact Tutorial: GCG & Factoring by Grouping
http://www.mathnotes.com/intermediate/Mchapter05/aw_MInterAct5_7.html

GCF & Factoring by Grouping Guide (.pdf file)
http://online.math.uh.edu/Math1300/ch4/s41/ex41.pdf

GCF & Factoring by Grouping
http://online.math.uh.edu/Math1300/ch4/s41/GCF/Lesson/Lesson.html

Factoring & Polynomials
http://www.okc.cc.ok.us/maustin/Factoring/Factoring.html

Factoring Strategies
http://hhh.gavilan.cc.ca.us/ybutterworth/intermediate/ch5Angel.doc