Algebra Connections

Mr. Breitsprecher's Edition | March 9, 2005 | Web: www.clubtnt.org/my_algebra

Dividing Polynomials

Easy does it! Let's look at how to divide polynomials by starting with the simplest case, dividing a monomial by a monomial. This is really just an application of the Quotient Rule that was covered when we reviewed exponents.

Next, we'll look at dividing a polynomial by a monomial. Lastly, we will see how the same concepts are used to divide a polynomial by a polynomial. Are you ready? Let's begin!

Review: Exponents and Polynomials

a^n means the product of *n* factors, each of which is *a* (i.e. $3^2 = 3*3$, or 9)

Exponent Rules

- **Product Role:** $a^m*a^n=a^{m+n}$
- **Power Rule:** $(a^m)^n=a^{mn}$
- **Power of a Product Rule**: $(ab)^n = a^nb^n$
- **Power of a Quotient Rule:** $(a/b)^n=a^n/b^n$
- **Quotient Rule:** $a^m/a^n=a^{m-n}$
- **Zero Exponent:** $a^0=1, a \neq 0$

Working with Polynomials

- **Adding Polynomials.** Remove parenthesis and combine like terms.
- **Subtracting Two Polynomials.** Change the signs of the terms in the second polynomial, remove parenthesis, and combine like terms.
- **Multiply Polynomials.** Multiply **EACH** term of one polynomial by **EACH** term of the other polynomial, and then combine like terms.

Special Products

- **FOIL** When multiplying binomials, multiply **EACH** term in the first binomial by **EACH** term in the second binomial. This multiplication of terms results in the pattern of **F**irst, **O**utside, **I**nside, and **L**ast.
- **Difference Of Squares:** $(a+b)(a-b) = a^2-b^2$
- **Perfect Square Trinomials:**
  - $(a+b)^2 = a^2+2ab+b2$
  - $(a-b)^2 = a^2 – 2ab + b^2$

Quotient of a Monomial by a Monomial

To divide a monomial by a monomial, divide numerical coefficient by numerical coefficient. Divide powers of same variable using the **Quotient Rule of Exponents** – when dividing exponentials with the same base we subtract the exponent on the denominator from the exponent on the numerator to obtain the exponent on the answer.

Examples:

- $(35x^3)/(7x) = 5x^2$
- $(16x^2 y^2)/(8xy^2) = 2x$

Remember: Any nonzero number divided by itself is one ($y^2/y^2 = 1$), and any nonzero number to the zero power is defined to be one (Zero Exponent Rule).

- $42x/(7x^3) = 6/x^2$

Remember: The fraction x/x^3 simplifies to $1/x^2$. The Negative Exponent Rule says that any nonzero number to a negative exponent is defined to be one divided by the nonzero number to the positive exponent obtained. We could write that answer as $6x^{-2}$.

Quotient of a Polynomial by a Monomial

To divide a polynomial by a monomial, divide each of the terms of the polynomial by a monomial. This is really an example of how we add fractions, recall:

$$\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$$

Think of dividing a polynomial by a monomial as rewriting each term of the polynomial over the denominator, just like we did when working with fractions. Then, we simplify each term.

Example:

- $(16x^3 - 12x^2 + 4x)/(2x)$
  $= (16x^3)/(2x)-(12x^2)/(2x)+(4x)/(2x)$
  $= 8x^2 - 6x + 2$

Quotient of a Polynomial by a Polynomial

To divide a polynomial by a polynomial, use long division, similar to the long division technique used in arithmetic. Remember in starting the long division process:

1. Write dividend and divisor in terms of descending powers of variable, leaving space for any missing powers of the variable or writing in the missing powers with coefficient zero. If there is more than one variable, arrange dividend and divisor in terms of descending order (from the term with the highest "degree" to the lowest "degree").
2. Divide first term of divisor into first term of dividend (On subsequent iterations, into first term of previous difference). Place this answer above long division symbol.
3. Multiply divisor by the expression just written above division symbol and align like terms.
4. Subtract line just written from line immediately above it. Remember to subtract we change the sign of the subtrahend and add (add the opposite).
5. Repeat steps 2 through 4 until the difference you obtain is a polynomial of degree less than the degree of the divisor.
6. If the final difference is zero, the division is exact. The quotient is the polynomial given across the top. If the difference in nonzero division is not exact, the quotient is the polynomial given across the top plus the remainder (polynomial in last line) divided by the divisor.

Example:

$$\begin{array}{r} 2x^2 + x + 3 \; R(-1)/(2x-1) \\ 2x+1 \overline{)\,4x^3 + 0x^2 + 5x - 4} \\ \underline{4x^3 - 2x^2} \\ 2x^2 - x \\ \underline{6x - 4} \\ 6x - 3 \\ \underline{} \\ -1 \end{array}$$

Source: http://home.sprynet.com

Polynomial Basics

Definitions

Monomial has one term: 5y or $-8x^2$ or 3.

Binomial has two terms: $-3x^2 + 2$, or $9y - 2y^2$

Trinomial has 3 terms: $-3x^2 + 2 + 3x$, or $9y - 2y^2 + y$

Degree Of The Term is the exponent of the variable: $3x^2$ has a degree of 2.

Degree of a Polynomial is the highest degree of any of its terms.

When the variable does not have an exponent -understand that there's a '1'.

One thing you will do when solving polynomials is combine like terms. Understanding this is the key to accurately working with polynomials. Let's look at some examples:

- Like terms: $6x + 3x - 3x$
- **NOT** like terms: $6xy + 2x - 4$

The first two terms are like and they can be combined:

$$5x^2 + 2x^2 - 3$$

Combining like terms, we get:

$$7x^2 - 3$$

Online Resources

How to Add, Subtract, Multiply, and Divide Polynomials
http://faculty.ed.umuc.edu/~swalsh/Math%20Articles/Polynomial.html

Dividing Polynomials

http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut35_div.htm

http://tutorial.math.lamar.edu/AllBrowsers/1314/DividingPolynomials.asp

Dr. Math: Dividing Polynomials.
http://mathforum.org/library/drmath/view/52901.html

Long Division of Polynomials.

http://www.sosmath.com/algebra/factor/fac01/fac01.html

http://www.purplemath.com/modules/polydiv2.htm

http://www.mathwords.com/p/polynomial_long_division.htm

http://www.math.utah.edu/online/1010/euclid/

http://tutorial.math.lamar.edu/AllBrowsers/1314/DividingPolynomials.asp

Polynomial Division and Factoring

http://campus.northpark.edu/math/PreCalculus/Algebraic/Polynomial/Factoring

Online Solutions. Scroll down this site to the form identified as **Division of polynomials: p(x) / f(x)**. It will divide polynomials to degree 6. Just set the coefficients, and click "**Divide**". Be sure that the degree of p(x) >= degree of f(x). Leading zero coefficients are ignored – use zeros for terms in the form that are not present in the problem you want to solve. http://www.egwald.com/linearalgebra/polynomials.php