Algebra Connections

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3 Methods, You Choose!

Factoring Trinomials: $ax^2 + bx + c$

Let's look at factoring trinomials with a "*leading coefficient other than 1*" We can write that form as $ax^2 + bx + c$, where a, b, and c are integers.

Like factoring any other polynomial, the first thing to do is to factor out all constants which evenly divide all three terms (Greatest Common Factor). If "*a*" is negative, factor out -1.

This will leave an expression in the form: $d(ax^2 + bx + c)$, where a, b, c, and d are integers, and $a > 0$. Now we have a factor (GCF) times a trinomial in the form $ax^2 + bx + c$. The next step is to factor the remaining trinomial. We will present 2 methods.

The first method, which will be referred to as "**Old School Algebra**." **NOTE THAT THIS IS NOT THE METHOD THAT IS PRESENTED IN OUR TEXTBOOK**. It is similar to the the method presented in our textbook (unit 4.3), but uses a different set of "tests" utilizing absolute values when "*c*" is positive and when "*c*" is negative.

It is presented here as an alternative perspective – **students need not study this method.** Like the method presented in our text, unit 4.3, it comes down to "*trial-by-error.*"

Our text presents a second, similar method, which I call "**Guess and By Golly**" (Unit 4.3). Like the "*Old School*" method, it involves determining possible factors and checking them until one works. Our text's method differs in how it looks at positive and negative values for "*c*." Rather than creating a set of rules about when "*c*" is positive and when "c" is negative, "*Guess and By Golly*" relies on an understanding of patterns with signs.

The third method, "**Factoring by Grouping**," (Unit 4.4 in our text) works by rewriting our trinomial into a 4 term polynomial that can be factored by groups.

Old School Algebra

Here is how to factor an expression $ax^2 + bx + c$, where $a > 0$ ("a" is positive):

1. Write out all the pairs of numbers that, when multiplied, produce "*a*" (i.e. $a_1*a_2=a$, $a_3*a_4=a$, etc.).
2. Write out all the pairs of numbers that, when multiplied, produce "*c*" (i.e. $c_1*c_2=c$, $c_3*c_4=c$, etc.).
3. Pick one of the a pairs, (a_1, a_2), and one of the c pairs, (c_1, c_2).
4. If $c > 0$ ("c" is positive): Compute $a_1c_1 + a_2c_2$. If $|a_1c_1 + a_2c_2| = b$, then the factored form of the quadratic is:
   - $(a_1x + c_2)(a_2x + c_1)$ if $b > 0$ ("b" is positive).
   - $(a_1x - c_2)(a_2x - c1)$ if $b < 0$ ("b" is negative).
5. If $a_1c_1 + a_2c_2 \neq b$, compute $a_1c_2 + a_2c_1$. If $a_1c_2 + a_2c_1 = b$, then the factored form of the quadratic is $(a_1x + c_1)(a_2x + c_2)$ or $(a_1x + c_1)(a_2x + c_2)$. If $a_1c_2 + a_2c_1 \neq b$, pick another set of pairs.
6. If $c < 0$ (is negative). Compute $a_1c_1 - a_2c_2$. If $|a_1c_1 - a_2c_2| = b$, then the factored form of the quadratic is $(a_1x - c_2)(a_2x + c_1)$ where $a_1c_1 > a_2c_2$ if $b > 0$ (is positive) and $a_1c_1 < a_2c_2$ if $b < 0$ (is negative).
7. Using **FOIL**, the outside pair plus (or minus) the inside pair must equal b. This is a check.

Example 1: $3x^2 - 8x + 4$

- Numbers that produce 3: (1, 3).
- Numbers that produce 4: (1, 4), (2, 2).

Continuing:

- (1, 3) and (1, 4): $1(1) + 3(4) = 11 \neq 8$. $1(4) + 3(1) = 7 \neq 8$.
- (1, 3) and (2, 2): $1(2) + 3(2) = 8$.
- $(x - 2)(3x - 2)$.
- Check: $(x - 2)(3x - 2) = 3x2 -2x - 6x + 4 = 3x2 - 8x + 4$.

Example 2: Factor $12x^2 + 17x + 6$

- Numbers that produce 12: (1, 12), (2, 6), (3, 4).
- Numbers that produce 6: (1, 6), (2, 3).

Continuing:

- (1,12) and (1,6): $1(1)+12(6) = 72$.

1(6) + 12(1) = 18.

- (1, 12) and (2, 3): 1(2) + 12(3) = 38. 1(3) + 12(2) = 27.
- (2, 6) and (1, 6): 2(1) + 6(6) = 38. 2(6) + 6(1) = 18.
- (2, 6) and (2, 3): 2(2) + 6(3) = 22. 2(3) + 6(2) = 18.
- (3, 4) and (1, 6): 3(1) + 4(6) = 27. 3(6) + 4(1) = 22.
- (3, 4) and (2, 3): 3(2) + 4(3) = 18. 3(3) + 4(2) = 17.
- (3x + 2)(4x + 3).
- **Check:** (3x + 2)(4x + 3) = $12x^2$ +9x + 8x + 6 = $12x^2$ + 17x + 6.

Example 3: Factor $4x^2$ - 5x - 21

- Numbers that produce 4: (1, 4), (2, 2).
- Numbers that produce 21: (1, 21), (3, 7).

Continuing:

- (1,4) and (1,21): 1(1)-4(21)= - 83. 1(21) - 4(1) = 17.
- (1,4) and (3,7): 1(3) - 4(7) = - 25. 1(7) - 4(3) = - 5.
- (x - 3)(4x + 7).
- Check: (x - 3)(4x + 7) = $4x^2$ +7x - 12x - 21 = $4x^2$ - 5x - 21.

Analysis: "Old School"

Probably, most will find this procedure complex – it requires a a different rule depending on the sign of "c." ***Mr. B's Algebra Connections*** **DOES NOT RECOMMEND STUDENTS USE THIS METHOD!**

It is actually the same process presented in our text, but it relies on a different set of rules using absolute values) depending on the sign of "*c*." Many will find different set of rules (depending on the sign of "*c*") frustrating – it seems like more to memorize.

For most of us, fewer rules are better. Our text uses the same concepts, but relies on an understanding of sign patterns when factoring.

The key is to remember: a negative number MUST have factors with different signs (one positive, the other negative). A positive number MUST have factors with the same signs (either both poitive or both negative) Seeing patterns in numbers is important in algebra. It is often easier to learn these if we allow someone to "guide" our practice.

Factor by Grouping (Unit 4.4)

1. Factor out the GCF, if it exists. This should always be your first step in EVERY factoring problem (no matter what method you choose to use).
2. Find 2 "mystery" numbers. These are similar to the numbers you sought when factoring simpler trinomials - but there is a slight difference. You still want the sum to be the x-coefficient (b), but now you want their product to equal ac, the product of the leading coefficient and the constant.
3. Replace the x-coefficient. Rewrite the polynomial, but where b once stood, write the sum of the two "mystery" numbers in parentheses. This is the key - we will now "blow this trinomial up" into two binomials.
4. Distribute x through the parentheses. There's still an x multiplied times the quantity in parentheses from step 3' multiply each of the mystery numbers by x to eliminate those parentheses. **AT THIS POINT WE HAVE A FACTOR BY GROUPING PROBLEM.**
5. Factor by Grouping.

Example: $6x^2$-x-12

Solution: This polynomial has no GCF. Skip right to calculating the "mystery" numbers; they should add to "b," in this case, -1 and have a product of 6 * (-12) = -72. Those numbers, therefore, must be -9 and 8. Replace the x-coefficient of -1 with the sum of those "mystery" numbers in parentheses (grouping symbols). This leaves us with:

$$6x^2+(-9+8)x-12$$

Distribute the *x* lying outside the parenthesis: $6x^2$-9x+8x-12

Now you can factor by grouping:

= 3x(2x-3) + (3x+4)
= (2x-3)(3x+4)

Analysis: Factor by Grouping

This procedure is direct and appears simpler than other methods. It is based on re-writing trinomials into 4 term polynomials (the opposite of simplifying) and is sometimes called "decomposition."

There is not as much "guess and by golly" or "trial and error" that is used in the textbook's method or the "old school" approach. Be careful about deciding whether The Bomb or our textbook's method is simpler. "Trial and error" might seem like extra work, but with practice, you will see patterns and choose the right solution more and more often.

Our textbook shows examples where we work out every possible solution **WRONG** before we find the right one. As we learn the "patterns," we will tend to start with the **RIGHT** solution without looking at all of the wrong ones. **THIS IS WHY GUIDED PRACTICE**, working with another person (like your professor or the Math Center), is important.

Please remember, text uses carefully selected problems to illustrate important patterns in numbers - patterns that reinforce algebra skills and concepts. Our textbook is a beautiful presentation of a complex subject. **Please practice Units 4.3 and 4.4**. This will build your algebra skills and then you can decide which to use.

Quick Review: "Guess and By Golly," Unit 4.3