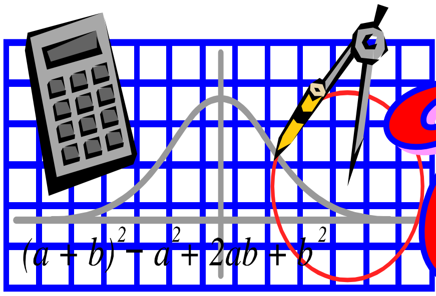

Algebra Connections

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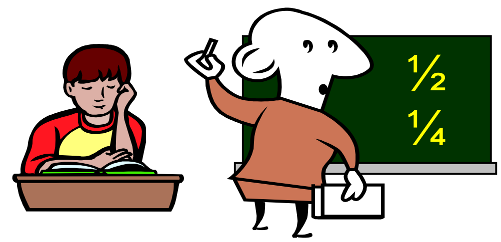
Fractions 101



A fraction has 2 main parts, the **numerator** (top) and **denominator** (bottom). The line between the two is called the "**fraction bar**." Need a little "trick" to keep this straight? Just think "N" for North (up), numerator; "D" for Down, denominator.

$$\frac{3}{4} = \frac{N}{D}$$

The denominator (down) or lower part of a fraction indicates how many equal portions or pieces of the whole there are. Note that when we divide a pizza into ½, each piece or half is of equal size.

The same is true if we cut a chunk of fudge into 4 pieces or ¼ slices. Each piece of fudge will be equal. The numerator (north or up) refers to how many of those equal pieces the fraction represents.

Types of Fractions

When we start dividing whole things into pieces, we have three situations: proper, improper, and mixed fractions. Let's review each.

Proper Fractions. The numerator is always smaller than the denominator. Most of us think of this type of fraction when thinking of parts of a whole. This type of fraction always represents less than 1. This is the simplest type of fraction to work with.

Examples: Proper Fractions

$$\frac{7}{8} \text{ or } \frac{5}{12} \text{ or } \frac{6}{17}$$

Improper Fractions. The numerator is always larger than the denominator. This fraction seems "top-heavy" or an odd way to express parts of a whole, because the fraction is actually greater than 1. The denominator (down) still tells us what size the equal parts are. The numerator (north or up) tells us that we have more than enough pieces to make a whole – this fraction always represents something greater than 1. This type of fraction can be useful to work with in many situations.

Examples: Improper Fractions

$$\frac{12}{7} \text{ or } \frac{5}{3} \text{ or } \frac{24}{7}$$

Mixed Fractions. Looking at improper fractions can be confusing, that's why we have mixed fractions. They clearly indicate how many parts of the whole there are with proper fraction and use a whole number to indicate how many complete "wholes" are. This is much easier for most of us to interpret than when looking at

Lowest Common Multiple = Lowest Common Denominator

The Lowest Common Multiple (LCM) is the smallest number that is a common multiple of two or more numbers. When working with fractions, this is also the Lowest Common Denominator (LCD). For example, the L.C.M of 3 and 5 is 15. The simple method of finding the L.C.M of smaller numbers is to write down the multiples of the larger number until one of them is also a multiple of the smaller number.

Example: Find the Lowest Common Multiple of 8 and 12.
Solution: Multiples of 12 are 12, 24... 24 is also a multiple of 8, so the L.C.M of 8 and 12 is 24.

Example: LCM/LCD of Big Numbers. Find all the prime factors of both numbers. Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included. (Note: this sounds different than our textbook; but if you think about it, it is really the same thing. We end up with prime factors from each number and use each prime factor the number of times it appears the most).

Example: Find the Lowest Common Multiple of 240 and 924.
Solution: Write the prime factorization of each. 924 = 2 x 2 x 3 x 7 x 11 and 240 = 2 x 2 x 2 x 2 x 3 x 5, therefore the lowest common multiple is:
(2 x 2 x 3 x 7 x 11) x (2 x 2 x 5) = 924 x 20 = 18,480

improper fractions.

Examples: Mixed Fractions

$$1\frac{2}{3} \text{ or } 7\frac{16}{37} \text{ or } 9\frac{1}{8}$$

Simplifying Fractions

The key to working with fractions is to express them as simply as possible. It is easiest to work with the smallest possible numerator (north – up) and denominator (down). This makes it easier to visualize and perform mathematical operations. A fraction is in its lowest terms when both the numerator and denominator cannot be divided evenly by any number except 1.

Equivalent Fractions. If the numerator and denominator of any fraction are multiplied by the same number, the fraction looks different but really represents the same part of the whole. We know that any single number multiplied by 1 is equal to the number we started with.

Note that any fraction that has the same numerator and denominator is equal to one – this is what we really have when we multiply the numerator and denominator by the same number.

We have equivalent fractions when we express a fraction in 2 different ways, both of which can be simplified to the same lowest terms. We also create an equivalent fraction when we divide the numerator and denominator by the same number – in this case, we are expressing the same fractions with smaller numbers.

Examples: Equivalent Fractions

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

Simplest Terms

Keeping fractions easy to look at, visualize, and work with is important. That is what we are doing when we simplify fractions. If we can divide both the numerator and denominator by the same number (just another way of expressing 1), we reduce the numbers used to express that fraction. This makes it simpler to work with.

When we reduce the numerator and denominator by dividing each by the same number and cannot reduce it any further, we have expressed that fraction in simplest terms.

Examples: Simplest Terms

$$\left(\frac{9}{12}\right) \div \left(\frac{3}{3}\right) = \frac{3}{4}$$

$$\left(\frac{48}{60}\right) \div \left(\frac{12}{12}\right) = \frac{4}{5}$$

Common Denominators

If we are comparing 2 different fractions (which is larger and which is smaller), we need to think of each fraction in terms of the same, equal parts of the whole (denominator). When we add or subtract fractions, we need to work with fractions that are expressed with the same denominator – a common denominator.

Rewriting fractions with the lowest common denominator is just a special example of writing an equivalent fraction – in this case, we find equivalent fractions that share the same denominator.

Finding the Common Denominator

Here is a step-by-step method for finding common denominators that will work every time:

1. Examine the fractions – can we see by observation what the common denominator is? If so – skip steps 2 and 3, go immediately to step 4. The more we practice working with fractions, the easier this step becomes.
2. If we did not see anything obvious in step 1, determine which fraction has the largest denominator.
3. Check to see if the smaller denominator divides into the larger one evenly. If so, move on to step 4. If not, check multiples of the larger denominator until you can find one that the smaller denominator can divide into evenly. If you remember our earlier lesson about lowest common multiples – that process will take you immediately to the lowest common denominator (LMC=LCD).
4. Write the 2 fractions as equivalent fractions with the common denominator.

Note that we can always find a common denominator by multiplying the denominators together – while this works, it can result in working with large numbers. Once this number is found, go to step 4. If you can comfortably work with these larger numbers, this method can be efficient.

Examples: Common Denominators

$$\left(\frac{1}{2} \,\&\, \frac{3}{7}\right) = \left(\frac{1}{2} \times \frac{7}{7}\right) \,\&\, \left(\frac{3}{7} \times \frac{2}{2}\right)$$

$$\frac{7}{14} \,\&\, \frac{6}{14}$$

$$\left(\frac{8}{15} \,\&\, \frac{1}{2}\right) = \left(\frac{8}{15} \times \frac{2}{2}\right) \,\&\, \left(\frac{1}{2} \times \frac{15}{15}\right)$$

$$\frac{16}{30} \,\&\, \frac{15}{30}$$

Improper Fractions & Mixed Fractions

While working with an improper fraction is no different than working with a mixed fraction, most of us will find it easier to interpret our results if we convert the improper fraction to a mixed fraction (whole number & proper fraction).

Divide the numerator (north = up) by the denominator (down) – the number of times the denominator goes evenly into the numerator is our "whole" number. The remainder, or parts that are left-over, are the numerator. Now we have are left with a fraction that represents less than one and a whole number for everything else.

$$\frac{11}{9} = 1\frac{2}{9}$$

Note: 11 divided by 9 is 1 with a remainder of 3.

$$\frac{402}{10} = 40\frac{2}{10}$$

Note: 402 divided by 10 is 40 with a remainder of 2.

$$\frac{32}{7} = 4\frac{4}{7}$$

Note: 32 divided by 7 is 4 with a remainder of 4.