Algebra Connections

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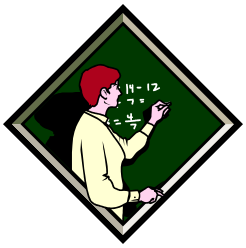
Working With Fractions



Understanding what fractions are is the key to being able to work with them. The rules of adding, subtracting, multiplying, and dividing them are the same whether we are working with fractions consisting of numbers or those containing variables.

Adding/Subtracting Fractions

Adding and subtracting fractions is not hard if we remember to work with common denominators. This should make sense. How can we make meaningful combinations of different parts of the whole if we are not talking about the same size parts?

For example, we don't speak of adding 1 pint to a quart of milk. We add 1 pint to 2 pints and end up with 3 pints. It would sound odd to talk about a "quart and a pint" of milk. Likewise, we don't add quarters and eighths without finding the common denominator first.

Be sure that **BEFORE** adding or subtracting fractions, we find a **COMMON DENOMINATOR.** Here is a 3-step process for adding/subtracting fractions:

1. Convert the fractions to equivalent fractions that have the same number in the denominator.
2. Add or subtract the numerators. Do not do anything with the denominator! Simply write your answer on the common denominator.
3. Simplify or reduce the fraction if possible.

Example: $\frac{1}{2}+\frac{3}{7}$

Note that the lowest common denominator is 14. Actually, any common denominator will do – but if we work with the lowest common denominator, we make simplifying our answer easier. We can find a common denominator by multiplying the original denominators together. For more on determining the LCD, please refer to the article to the left.

This means we will need to multiply BOTH the numerator and the denominator of the first fraction by 7. The numerator and denominator of the second fraction will need to be multiplied by 2.

$$\left(\frac{1}{2}\times\frac{7}{7}\right)+\left(\frac{3}{7}\times\frac{2}{2}\right)=$$

$$\frac{7}{14}+\frac{6}{14}=\frac{13}{14}$$

Lowest Common Denominators & Equivalent Fractions

Because we can only add or subtract fractions if they represent the same pieces of the whole (same denominators), finding the lowest common denominator (LCD) is important. The lowest common denominator is also the **least common multiple** – this was reviewed earlier. Here is a short review on how to find the LCD (which is the LCM), so that we can create equivalent fractions that maintain their original identify.

1. Factor each denominator into its prime factors
2. Build the LCD
   a. Each unique prime factor must appear at least once
   b. Raise factors to the highest power it appears in any one of the original denominators
3. Convert fractions INDIVIDUALLY to equivalent fractions with the new LCD by multiplying the numerator AND denominator by the missing factors

While this procedure may, at first, sound different than the process that was introduced earlier in the semester when we reviewed least common multiples, do you see it is really just a simpler set of steps to accomplish the same thing. The least common multiple IS the lowest common denominator!

Example: $\left(\frac{8}{15}-\frac{1}{2}\right)$

The lowest common denominator here is 30 – it happens to be the product of the 2 different denominators we started out with. This will always work, but when we start out with larger numbers in the denominators, it becomes more cumbersome to work with.

We will need to multiply **BOTH** the numerator and the denominator of the first fraction by 2. The numerator and denominator of the second fraction will need to be multiplied by 15.

$$\left(\frac{8}{15}\times\frac{2}{2}\right)-\left(\frac{1}{2}\times\frac{15}{15}\right)=$$
$$\frac{16}{30}-\frac{15}{30}=\frac{1}{30}$$

Note that when adding or subtracting mixed fractions, we can add/subtract each part by itself (whole number and fraction) and write the answer together. The key is to be sure the fractional part has a common denominator.

Example: $1\frac{2}{3}+2\frac{1}{6}$

Adding the whole number part of these mixed fractions is straightforward (1+2=3). To add the fraction part, we need to find a common denominator – the **lowest common denominator** (LCD) is 12.

We will need to multiply **BOTH** the numerator and the denominator of the first fraction by 4. The numerator and denominator of the second fraction will need to be multiplied by 2. Adding the fraction part (using the common denominator):

$$\left(\frac{2}{3}\times\frac{4}{4}\right)+\left(\frac{1}{6}\times\frac{2}{2}\right)=$$
$$\frac{8}{12}+\frac{2}{12}=\frac{10}{12}=\frac{5}{6}$$

Now we can combine our 2 parts (whole and fraction) to get:

$$3\frac{5}{6}$$

Multiplying Fractions

In many ways, multiplying fractions is the easiest operation of the four. There is no need to have a common denominator. Be careful with mixed fractions, however. Before multiplying them, re-write them as an improper fraction. You can always express your final answer back to a mixed fraction, if need be.

Here is a 3-step process for multiplying fractions:

1. Change all mixed fractions to improper fractions
2. Multiply the numerators together and multiply the denominators together. Each results in the numerator/denominator of your answer.
3. Simplify or reduce the fraction if possible.

Example: $\frac{3}{8}\times\frac{2}{7}=\frac{6}{56}\ or\ \frac{3}{28}$

Dividing Fractions

If one understands how to multiply fractions, dividing them is not hard. Recall that we can define division in terms of multiplication – instead of dividing by a number; we can multiply by the reciprocal. This is true in EVERY case – but it is essential when dividing fractions. Here is a 3-step process for dividing fractions:

1. Change all mixed numbers to improper fractions.
2. Flip the second fraction, placing the bottom number on top and the top number on the bottom.
3. Continue as with the multiplication of fractions.

Example: $\frac{5}{12}\div\frac{2}{9}$

$$=\frac{5}{12}\times\frac{9}{2}=\frac{45}{24}\ or\ \frac{15}{8}$$

When dividing fractions, note that we have really just taken advantage of our definition for division. Thinking of division in terms of multiplication by the reciprocal and thinking of subtraction in terms of addition of the opposite (change sign) can be useful.

Note that addition and multiplication share a special feature: order does not matter (commutative and associative properties of addition/multiplication). This means that when we are adding or multiplying fractions, we can work with more than 2 at a time.

Decimals = Fractions

We should close our review of fractions by pointing out that decimals are really just fractions where he indicated division has been performed. They are really the same as fractions with denominators of 10, 100, 1,000, and so forth.

The decimal point really indicates that we are dealing with a "part-of-a-whole" or a fraction. Because the denominators of these fractions are always a multiple of ten, it is easier to indicate this with a decimal point.

Example: $12.31=12\frac{31}{100}$

More Fraction Resources

Fractions, Decimals, Percentages: Explanations
http://descartes.cnice.mecd.es/ingles/3rd_year_secondary_educ/Fract_dec_pt ges/Fracciones_1.htm

Simplifying Fractions
http://www.helpwithfractions.com/simplifying-fractions.html

Fraction Links
http://www.mathleague.com/help/fractions/fractions.htm

Reducing Fractions & LCD
http://www.sparknotes.com/math/prealgebra/fractions/section2.rhtml

Fractions Fast Facts
http://www.mccc.edu/~kelld/fraff.htm

PowerPoint Presentation: Finding the Lowest Common Denominator
http://www.ceres.k12.ca.us/iweb/lessons/Monica's%20Math/Fractions%20XII.ppt