Algebra Connections

Mr. Breitsprecher's Edition November 14, 2005 Web: www.clubtnt.org/my_algebra

Lines, Equations & Inequalities

A graph of a straight line is called a **linear equation in 2 variables**. They can always be written in the form Ax+By = C where A and B are not BOTH 0. Note that a linear equation may appear to have only one variable (x or y). This means that the other variable has a coefficient of 0. We call the form Ax+By = C **standard form**.

A linear equation defines a relationship between 2 variables, x and y, and is called a relation. The set of all x-coordinates is called the domain of the relation. The set of all y-coordinates is called the range of the relation.

A **function** is a set of ordered pairs that assigns each x-value precisely one y-variable. In other words, each x-value predicts a y-value.

When setting up linear equations, it is important establish the relationship so that each value of y is dependent on the value of x. This results in each value of y being a "function" of x.

Function notation uses the symbol *f(x)* means function of x. for example: *f(x)* = 3x-7.

$f(-1) = 3(-1)-7 = -10$

In other words, f(-1) means to substitute that value (-1) for x and determine the corresponding y-value.

Because function notation means that each x-value has precisely 1 y-value, there can only be 1 value of y for each value of x. In other words, a vertical line can always be drawn through and it will only intersect a linear equation 1 time – this is called the **vertical line test**. A function cannot curve back on itself or contain any type of angle that results in a value of x to "predict" more than 1 y.

What Does Slope Mean

Slope tells us the slant or tilt of a line – it is defined as **rise over run** and can be expressed as:

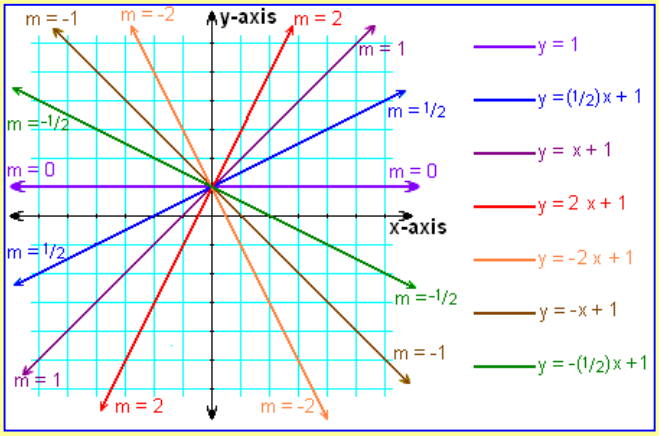
$$m = \frac{\text{rise}}{\text{run}} \qquad \frac{\text{change in y}}{\text{change in x}} \qquad \frac{y_2 - y_1}{x_2 - x_1}$$

Note that when we take an equation in standard form (Ax+By = C) and solve it for y, the resulting coefficient of x is the slope – THIS IS NOT THE SAME AS SAYING THAT" A" IS THE SLOPE – IT CERTAINLY IS NOT!

Please see the example below:

10x-5y = (-5)
-10x+10x-5y = (-5)–10x
-5y = (-10)-10x
(-5y)/(-5) = [(-5)-2x]/(-5)
y = 1+2x
y = 2x+1

Note that we now have a slope of 2, which is not the coefficient A from our standard form.



When we solve any equation in standard form, the resulting coefficient of x is actually the slope of the line. Please see the examples above.

Three Useful Forms for Linear Equation in 2 Variables

We will use 3 forms to represent linear equations in 3 variables:

1. **Standard Form:** Ax+By = C, where A and B are not both 0.
2. **Slope-Intercept Form:** y = mx+b, where m is the slope of the line and b is the y-intercept (0, b).
3. **Point-Slope Form:** $y-y_1 = m(x-x_1)$, where m is the slope and x_1, y_1 is a point on the line.

Note that standard form is the most general equation – recall that the multiplication property of equality tells us we can multiply BOTH SIDES of an equation in standard from by some number and not change the identity of the equation (line).

In other words, A, B, and C will change and each will still be valid. For this reason, standard form has limited practical applications.

The slope-intercept form, however, will always have unique values of m (slope) and b (x-intercept) for each line.

Important Definitions

Rectangular Coordinate System. Plane with vertical and horizontal number lines that intersect at their 0 coordinate.

- ✓ **X-axis.** The number horizontal line
- ✓ **Y-axis.** The number vertical line
- ✓ **Origin.** Where the lines cross, represented by the point x=0, y=0

Ordered Pair. A pair of coordinates, one representing "x" and one representing "y." By convention, written in the format of (x,y).

Relation. Set of ordered pairs

Domain. Set of all x-coordinates

Range. Set of all y-coordinates

Function. Set of ordered pairs that assigns to each x-value exactly one y-value

Vertical Line Test. If a vertical line intersects a graph more than once, the graph is not a function

Online Resources

The Coordinate Plane and Graphing Linear Equations. In this unit we'll be learning about equations in two variables. A coordinate plane is an important tool for working with these equations. http://www.math.com/school/subject2/lessons/S2U4L1GL.html

Tutorial: Graphing Linear Equations. When you graph linear equations, you will end up with a straight line. Let's see what you can do with these linear equations. http://www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut21_graph.htm

Graph of a Line. We can graph the linear equation defined by y = x + 1 by finding several ordered pairs. For example, if x = 2 then y = 2 + 1 = 3, giving the ordered pair (2, 3). Also, (0, 1), (4, 5), (-2, -1), (-5, -4), (-3, -2), among many others, are ordered pairs that satisfy the equation. http://www.algebra-online.com/graph-lines-1.htm

Graphs of Linear Equations: Lines and Slope. Lessons, some practice sets http://www.math.com/school/subject2/lessons/S2U4L1GL.html

Graphing Inequalities in 2 Variables. The solution set for an inequality in two variables contains ordered pairs whose graphs fill an area on the coordinate plane called a half-plane. An equation defines the boundary or edge of the half-plane. http://www.algebra-online.com/graphing-inequalities-1.htm

Worksheets: Linear Equations and Inequalities. Need more practice? These worksheets will help. http://www.edhelper.com/LinearEquations.htm

ThinkQuest: Graphine Linear Equations and Inequities. Let's review basics of graphic equations and inequalities. http://library.thinkquest.org/10030/6gleai.htm

Create Graphs of Equations and Inequalities

Automatic Graph Solutions: Equations. Enter the equation you want to plot, in terms of the variables x and y, set the limits and click the Plot button. http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=quickmath&s1=graphs&s2=equations&s3=basic

Automatic Graph Solutions: Inequalities. Enter the polynomial inequality you want to plot, in terms of the variables x and y, set the limits and click the Plot button. http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=quickmath&s1=graphs&s2=inequalities&s3=basic

Likewise, the point-slope form will always have a unique value for m (slope) and x_1, y_1 must be points on the line.

Graphing Linear Inequalities in Two Variables

A linear inequality is two variables is an inequality that can be written in one of the following forms:

- ✓ $Ax+By < C$
- ✓ $Ax+By \leq C$
- ✓ $Ax+By > C$
- ✓ $Ax+By \geq C$

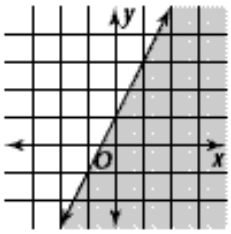
The solution set for an inequality in two variables contains ordered pairs whose graphs fill an area on the coordinate plane called a **half-plane**. An equation defines the **boundary** or edge of the half-plane.

Graphing Linear Inequalities

1. Graph the boundary line by graphing the related equation. Draw the line solid if the inequality symbol is $\leq$ or $\geq$. Draw the line dashed if the inequality is $<$ or $>$.
2. Choose a test point not on the line. Substitute its coordinates into the original inequality.
3. If the resulting inequality is true, shade the half-plane that contains the test point. If the inequality is not true, shade the half-pane that does not contain the test point.

Example: $y-2x \leq 1$

The boundary would be $y = 2x+1$. Choose easy to work with values for x and find corresponding y-values, i.e. (0, 3), (1, 3), and (3, 7). Remember, 2 points determine a line. The third point is a "check." We would graph this as a solid line; our boundary is included in our half-plane.



To determine which half-plane, represents our solution, choose a test point; say (0, 0). Substituting this back in our original inequality:

$y-2x \leq 1$, we see $0-2(0) \leq 1$ results in $0 \leq 1$. This is a true statement, so we shade that half-plane.