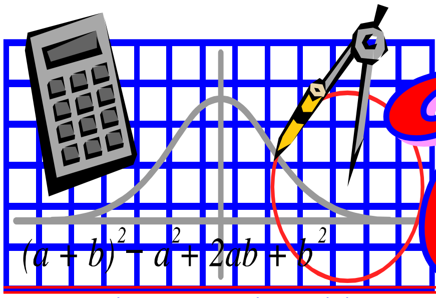
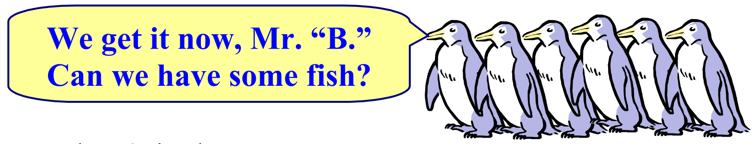


Algebra Connections

Mr. Breitsprecher's Edition **January 26, 2005** **Web: www.clubtnt.org/my_algebra**

Negative Numbers



Although you may not realize it, you use the set of **integers** in everyday mathematical calculations. Positive integers are all the whole numbers greater than zero: 1, 2, 3, 4, 5, ...

Negative integers are all the opposites of these whole numbers: -1, -2, -3, -4, -5, … . We do not consider zero to be a positive or negative number. For each positive integer, there is a negative integer, and these integers are called opposites. For example, -3 is the opposite of 3, -21 is the opposite of 21, and 8 is the opposite of -8. If an integer is greater than zero, we say that its sign is positive. If an integer is less than zero, we say that its sign is negative.

Integers are useful in comparing a direction associated with certain events. Suppose I take five steps forwards: this could be viewed as a positive 5. If instead, I take 8 steps backwards, we might consider this a -8. Temperature is another way negative numbers are used. On a cold day, the temperature might be 10 degrees below zero Celsius, or -10°C. (note: we used Celsius, not Fahrenheit, because Celsius has an absolute zero, water freezes at 0; Fahrenheit uses 32 for the same point).

The Number Line

The number line is a line labeled with the integers in increasing order from left to right, that extends in both directions (Please see top heading):

For any two different places on the number line, the integer on the right is greater than the integer on the left.

Example:

9 > 4, 6 > -9, -2 > -8, and 0 > -5

Absolute Value of an Integer

The number of units a number is from zero on the number line. The absolute value of a number is always a positive number (or zero). We specify the absolute value of a number n by writing n in between two vertical bars: |n|.

Examples:

|6| = 6
|-12| = 12
|0| = 0
|1234| = 1234
|-1234| = 1234

Adding Integers

When adding integers of the same sign, we add their absolute values, and give the result the same sign.

(Continued on page 2)

Let's Ask Dr. Math

What are some good ways to explain negative numbers:? Here are some "real-world" examples:

Accounting. Say I borrow $5 from you and buy lunch since I forgot my wallet. I owe you $5 so on my balance sheet I have -5. Later I get my wallet and pay you the $5 back so I can subtract my debit (a net credit, subtracting a negative is a positive). So we have:

-5 -(-5) = -5 +5 = 0, which is what I now owe.

Driving. You are driving with cruise control set at 65mph (in a 65 zone, of course), which we will call your reference speed. You see a sign stating that you are entering a 55 zone so you slow down 10 mph (-10). After a few miles a new sign informs you that you are entering a 65 zone again so you resume your original speed, thus removing (subtracting) the -10mph modification. We thus have -10 - (-10) = 0, or no speed modification (thus you are moving at the reference speed of 65 again).

Books. You borrow 3 books from a library. You thus owe three books (-3). You read one and return (subtract) it (a borrowed book is a minus, thus a -1) and thus you have subtracted one book you owe, and now owe only two. And we have:

-3 -(-1) = -3 + 1 = -2

Behavior. Johnny swears and fights a lot (two negatives). He feels he wants to get better so he decides to stop (thus removing or subtracting) fighting (a negative). Thus he now has -2 - (-1) = -2 + 1 = -1, or 1 negative behavior he does a lot.

Remember: Subtracting a negative is adding, a positive. Subtraction can ALWAYS be though of as adding the opposite. If the greater of the 2 numbers is positive, so it the answer. If the greater of the 2 numbers is negative, the answer is negative. For example, (the opposite).

Source: mathforum.org

Practice: Here's an interactive Web: http://www.quia.com/fc/22424.html

Adding Integers *(from page 1)*

Examples:

2 + 5 = 7
(-7) + (-2) = -(7 + 2) = -9
(-80) + (-34) = -(80 + 34) = -114

When adding integers of the opposite signs, we take their absolute values, subtract the smaller from the larger, and give the result the sign of the integer with the larger absolute value.

Example:

8 + (-3) = ?

The absolute values of 8 and -3 are 8 and 3. Subtracting the smaller from the larger gives 8 - 3 = 5, and since the larger absolute value was 8, we give the result the same sign as 8, so 8 + (-3) = 5.

Example:

8 + (-17) = ?

The absolute values of 8 and -17 are 8 and 17. Subtracting the smaller from the larger gives 17 - 8 = 9, and since the larger absolute value was 17, we give the result the same sign as -17, so 8 + (-17) = -9.

Example:

-22 + 11 = ?

The absolute values of -22 and 11 are 22 and 11. Subtracting the smaller from the larger gives 22 - 11 = 11, and since the larger absolute value was 22, we give the result the same sign as -22, so -22 + 11 = -11.

Example:

53 + (-53) = ?

The absolute values of 53 and -53 are 53 and 53. Subtracting the smaller from the larger gives 53 - 53 =0. The sign in this case does not matter, since 0 and -0 are the same. Note that 53 and -53 are opposite integers. All opposite integers have this property that their sum is equal to zero. Two integers that add up to zero are also called additive inverses.

Subtracting Integers

Subtracting an integer is the same as adding its opposite.

Examples:

In the following examples, we convert the subtracted integer to its opposite, and add the two integers.

7 - 4 = 7 + (-4) = 3
12 - (-5) = 12 + (5) = 17
-8 - 7 = -8 + (-7) = -15
-22 - (-40) = -22 + (40) = 18

Note that the result of subtracting two integers could be positive or negative.

Multiplying Integers

To multiply a pair of integers if both numbers have the same sign, their product is the product of their absolute values (their product is positive). If the numbers have opposite signs, their product is the opposite of the product of their absolute values (their product is negative). If one or both of the integers is 0, the product is 0.

Examples:

In the product below, both numbers are positive, so we just take their product.

4 × 3 = 12

In the product below, both numbers are negative, so we take the product of their absolute values.

(-4) × (-5) = |-4| × |-5| = 4 × 5 = 20

In the product of (-7) × 6, the first number is negative and the second is positive, so we take the product of their absolute values, which is |-7| × |6| = 7 × 6 = 42, and give this result a negative sign: -42, so (-7) × 6 = -42.

In the product of 12 × (-2), the first number is positive and the second is negative, so we take the product of their absolute values, which is |12| × |-2| = 12 × 2 = 24, and give this result a negative sign: -24, so 12 × (-2) = -24.

To multiply any number of integers:

1. Count the number of negative numbers in the product.
2. Take the product of their absolute values.
3. If the number of negative integers counted in step 1 is even, the product is just the product from step 2, if the number of negative integers is odd, the product is the opposite of the product in step 2 (give the product in step 2 a negative sign). If any of the integers in the product is 0, the product is 0.

Example:

4 × (-2) × 3 × (-11) × (-5) = ?

Counting the number of negative integers in the product, we see that there are 3 negative integers: -2, -11, and -5. Next, we take the product of the absolute values of each number:

4 × |-2| × 3 × |-11| × |-5| = 1320.

Since there were an odd number of integers, the product is the opposite of 1320, which is -1320, so:

4 × (-2) × 3 × (-11) × (-5) = -1320.

Dividing Integers

To divide a pair of integers if both integers have the same sign, divide the absolute value of the first integer by the absolute value of the second integer.

To divide a pair of integers if both integers have different signs, divide the absolute value of the first integer by the absolute value of the second integer, and give this result a negative sign.

Examples:

In the division below, both numbers are positive, so we just divide as usual.

4 ÷ 2 = 2.

In the division below, both numbers are negative, so we divide the absolute value of the first by the absolute value of the second.

(-24) ÷ (-3) = |-24| ÷ |-3|=24÷3=8.

In the division (-100) ÷ 25, both number have different signs, so we divide the absolute value of the first number by the absolute value of the second, which is |-100| ÷ |25| = 100 ÷ 25 = 4, and give this result a negative sign: -4, so (-100) ÷ 25 = -4.

In the division 98 ÷ (-7), both number have different signs, so we divide the absolute value of the first number by the absolute value of the second, which is |98| ÷ |-7| = 98 ÷ 7 = 14, and give this result a negative sign: -14, so 98 ÷ (-7) = -14.

Source: *www.mathleague.com*