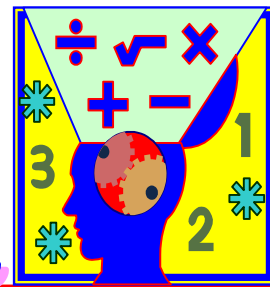


Algebra Connections

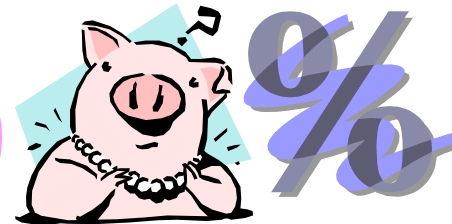


Mr. Breitsprecher's Edition

January 26, 2005

Web: www.clubtnt.org/my_algebra

Understanding Percents



Percents, like fractions and decimals, are way to express how a part of something relates to a whole. If we talk about $1/2$, $.5$, or 50% of a pizza, we are talking about one of 2 equal parts – half the pizza.

Percentages describe the relationship to a whole divided into 100 parts. The word “percent” means “out of 100.” The whole part is considered 100%. Do you see that the percent sign (%) consists of 2 zeros, as in 100?

Percentages can also be used to represent more than the whole amount (100%). Assume we have 100 pieces of candy. That whole represents 100%. Let’s assume we buy 50 more pieces of candy – 50 parts of 100 equals 50%. Notice that our original $100\% + 50\%$ more = 150% of what we started with.

The advantage of working with percentages is that we express each part of the whole in terms of $1/100$. Percentages allow us to make

meaningful comparisons between parts of a whole, because they always refer to parts out of 100. If we convert fractions to percentages (by performing the indicated division and multiplying by 100), we can now easily see the relationship between fractions – even when the denominators are different.

A percentage is really just a decimal multiplied by 100. Most of us will find it easier to work with percentages, because we don’t have to deal with as many decimal places (i.e. $12.5\% = .125$).

The Three Percentage Cases

To explain the cases that arise in problems involving percents, it is necessary to define the terms that will be used. **Rate (r)** is the number of hundredths parts taken. This is the number followed by the percent sign. The **base (b)** is the whole on which the rate operates. **Percentage (p)** is the part of the base determined by the rate. In the example:

$$5\% \text{ of } 40 = 2$$

- ▶ 5% is the rate (r)
- ▶ 40 is the base (b)
- ▶ 2 is the percentage. (p)

There are three cases that usually arise in dealing with percentage, as follows:

Case I. Find the percentage when the base and rate are known, i.e. What number (p) is 6% (r) of 50 (b)?

Case II. Find the rate when the base and percentage are known, i.e. 20 (p) is what percent (r) of 60 (b)?

Case III. Find the base when the percentage and rate are known, i.e. the number 5(p) is 25% (r) of what number (b)?

Source: <http://www.tpub.com/math1/7a.htm>

Common Equivalents

Here are some common equivalents. Do you see the patterns when you relate one column to another?

	Fraction	Decimal	Percentage
Twentieths	$1/20$.05	5%
Tenths	$1/10$.10	10%
	$3/10$.30	30%
	$7/10$.70	70%
	$9/10$.90	90%
Eighths	$1/8$.125	12.5%
	$3/8$.375	37.5%
	$5/8$.625	62.5%
	$7/8$.875	87.5%
Sixths	$1/6$.16666	16 $2/3\%$
	$5/6$.83333	83 $1/3\%$
Fifths	$1/5$.2	20%
	$2/5$.4	40%
	$3/5$.6	60%
	$4/5$.8	80%
Quarters	$1/4$.25	25%
	$3/4$.75	75%
Thirds	$1/3$.33333	33 $1/3\%$
	$2/3$.66666	66 $2/3\%$
Half	$1/2$.5	50%

Case I. 6% of 50 = ? The "of" has the same meaning as it does in fractional examples, such as $1/4$ of $16 = 4$

In other words, "of" means to multiply. Thus, to find the percentage, multiply the base by the rate. Of course the rate must be changed from a percent to a decimal before multiplying can be done. **To find the percentage, multiply the Rate times base and then divide by 100.**

$$\begin{aligned} [6\% (r) \text{ of } 50 (b)]/100 &= p \\ (6 * 50)/100 &= 3 \end{aligned}$$

The number that is 6% of 50 is 3.

To explain **Case II and Case III**, we notice in the foregoing example that the base corresponds to the multiplicand, the rate corresponds to the multiplier, and the percentage corresponds to the product.

$$50 (b) * .06 (r) = 3.00 (p)$$

Case II, ?% of 60 = 20. Recalling that the product divided by one of its factors gives the other factor, we can solve the following problem:

$$?\% \text{ of } 60 = 20$$

We are given the base (60) and percentage (20).

$$60 (b) * ? (r) = 20 (p)$$

We then divide the product (percentage) by the multiplicand (base) to get the other factor (rate). Percentage divided by base times 100 equals rate. The rate is found as follows:

$$20/60 * 100 = 33 \frac{1}{3} \%$$

Case III, 25% of ? = 5. The rule for Case II, as illustrated in the foregoing problem, is as follows: To find the rate when the percentage and base are known, divide the percentage by the base and multiply by 100.

The unknown factor in **Case III** is the base, and the rate and percentage are known.

$$25\% \text{ of } ? = 5$$

We are given the rate (25) and percentage (5). To find the base when the percentage and rate are known, divide the percentage by the rate and multiply by 100.

$$5/25 * 100 = 20$$

Another Perspectives: Ratios, Proportions & Percents Cross-Products to the Rescue!!!

Sometimes a different point of view is helpful – it gives us choices. An alternative way to think of percentages is to understand the relationship between ratios, proportions, and percentages.

Ratios tell how one number is related to another number. A ratio may be written as A:B or A/B or by the phrase "A to B". A ratio of 1:5 says that the second number is five times as large as the first. Percentages are really all ratios – just remove the “%” sign and write the remaining number over 100. For example, 25% equals “25 to 100.” or the ratio 25/100.

A proportion is a mathematical statement that two ratios are equal. If the two ratios are not equal, then it is not a proportion. An example would be the proportion that compares $3/6 = 4/8$. Do you see that each is an equivalent fraction (in this example, $1/2$). Please look at the proportion above and notice that the product of the 2 outside terms equals the product of the 2 inside terms, i.e. $(3*8)=(4*6)$. This will always be true of any proportion. This is called a “**cross-product**.”

If we remember that all percentages can be expressed as a ratio by removing the “%” and placing the remaining number over 100, then we can use cross-products to solve the 3 types of percentage problems we have been discussion. Many will find this approach easier; because if we set up our proportions correctly and understand how to algebraically solve for the missing variable, then we do not need to consider each of the three cases discussed in the preceding article. Cross-products will work for all three!

Case I, 6% of 50 = ? Realize that we are solving for the percentage. 6% (rate or “r”) can ALWAYS be expressed as the ratio 6/100. The base is 50. We can use a variable (p) to represent the question mark (percentage) and write that ratio as p/50. ALWAYS express this ratio as the percentage over the base. Now we have the following proportion: $6/100 = p/50$. The cross products are $(6*50) = (100*p)$, the product of the outside terms equals the product of the inside terms. This will simplify to $300 = 100p$. Solving for “p”, we get $p=300/100$ or 3. This is the same percentage that we calculated earlier.

Case II, ?% of 60 = 20. Realize that we are solving for the rate (r). It can be expressed as the ratio as r/100 (always rate/100). We are given the percentage (20) and the base (60). These are expressed as the ratio 20/60 (always percentage over base). Now we have the following proportion: $r/100=20/60$. The cross products are $(r* 60) = (100*20)$ When proportion is expressed in this format, the cross-product is always the product of the outside terms equals the product of the inside terms. This can be simplified, $r(60) = 2000$. Solving for rate (r), we get $r = 2000/60$ or $33 \frac{1}{3}\%$. This is the same rate we calculated earlier.

Case III, 25% of ? = 5. Realize that we are solving for the base. We can express the rate as the ratio 25/100 (always as rate/100). We are given the percentage (and are solving for the base). This can be expressed as the ratio 5/b (always percentage/base). Now we have the proportion of $(25/100) = (5/b)$. The cross product is $(25*b) = (100*5)$. When proportion is expressed in this format, the cross-product is always the product of the outside terms equals the product of the inside terms. Solving for b, we get $b = (100*5)/25$ or 20. This is the same base we calculated earlier.