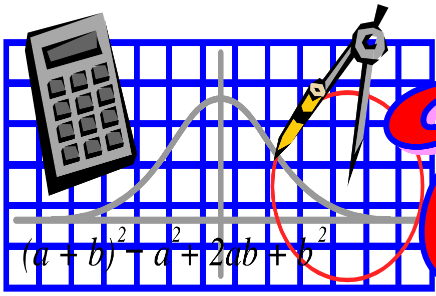


# Algebra Connections


Mr. Breitsprecher's Edition | March 29, 2005 | Web: www.clubtnt.org/my_algebra

# Solving Quadratic Equations by Factoring

Solving equations is an essential part of Algebra. Factoring equations breaks mathematical statements that look complex into smaller parts. Often, what looks large, complex and difficult becomes clear when we can look at it in smaller or simpler terms.

A **polynomial** is a finite sum of terms of the form $ax^n$ where $a$ is a real number and $n$ is a while number. Example: $5x^3-6x^2+3x-6$

A **quadratic equation** is an equation that can be written in the form $ax^2+bx+c=0$ with $a$ not equal to 0. When a quadratic equation is written in the form $ax^2+bx+c=0$, it is called **standard form.**

If we can write a polynomial in this standard form, it is a quadratic equation; $2x^2+5x = -3$ is an example of a polynomial equation. Here, the same equation is written in standard form:

$$2x^2+5x+3=0$$

While the polynomial we started with did not look like a quadratic equation, it is. Note that the degree of the above a quadratic equation is 2. This is a common type of equation.

### Zero Factor Property

If $ab = 0$, then $a = 0$ or $b = 0$. Zero is the key, because the only way a product can become 0 is if at least one of its factors is 0. We would not be able to make a general statement about the factors if the product was set equal to any other number. For example, if $ab = 1$, then $a = 7$ and $b = 1/7$ or $a = 4$ and $b = 1/4$, etc. But with the product set equal to 0, we can guarantee finding the solution by setting each factor equal to 0. This is why standard form ($ax^2+bx+c=0$) is so important.

### Solving a Quadratic Equations by Factoring

**Step 1:** Write the equation in standard form. First, if necessary, simplify the equation (i.e. clear any fractions or parenthesis () and combine like terms **BEFORE** rewriting in standard form)
**Step 2:** Factor completely
**Step 3:** Set each factor containing a variable equal to 0.
**Step 4:** Solve the resulting equations.

**Example:** $3x^2 = 13x-4$

**Step 1.** Rewrite in standard form: $3x^2-13x+4 = 0$
**Step 2.** Factor completely: $(3x-1)(x-4) = 0$
**Step 3.** Set each factor containing a variable equal to 0: $3x-1 = 0$ and $x+4 = 0$
**Step 4.** Solve the resulting equations:

| | |
|---|---|
| $3x-1 = 0$ | $x-4 = 0$ |
| $3x = 0+1$ | $x = 0+4$ |
| $x = 1/3$ | $x = 4$ |

In this problem, there is no simplifying – we were able to immediately rewrite in standard form by using the addition property of equality.

**Example:** $x^2 = 121$

**Step 1.** Rewrite in standard form: $x^2-121 = 0$. ***NOTE:*** *This is a difference of squares;*

$$a^2-b^2 = (a+b)(a-b)$$

**Step 2.** Factor completely:

$$(x-11)(x+11) = 0$$

**Step 3.** Set each factor containing a variable equal to 0: $x-11 = 0$ and $x+11 = 0$
**Step 4.** Solve the resulting equations:

| | |
|---|---|
| $x-11 = 0$ | $x+11=0$ |
| $x = 11$ | $x = -11$ |

being able to recognize **difference of squares** and **perfect square** trinomials is important.

**Example:** $x^2+12x = -36$

**Step 1.** Rewrite in standard form: $x^2+12+36 = 0$. ***NOTE:*** *This is a perfect square trinomial with a positive middle term;*

$$a^2+2ab+b^2 = (a+b)^2$$

**Step 2.** Factor completely:

$$(x+6)^2 = 0$$

**Step 3.** Set each factor containing a variable equal to 0: $x+6 = 0$. Note: In a perfect square trinomial, there is only 1 factor
**Step 4.** Solve the resulting equations:

$$x + 6 = 0$$
$$x = -6$$

Because the first term and coefficient were perfect squares and the middle term was positive, **if this trinomial is factorable**, we have a perfect square trinomial that is the sum of two terms.

**Example:** $4m^2-4m = -1$

**Step 1.** Rewrite in standard form: $4m^2-4m+1 = 0$. ***NOTE:*** *This is a perfect square trinomial with a negative term;* $a^2-2ab+b^2 = (a-b)^2$

**Step 2.** Factor completely: $(2x-1)^2 = 0$

**Step 3.** Set each factor containing a variable equal to 0: $2x-1 = 0$. ***Note:*** *In a perfect square trinomial, there is only 1 factor.*

**Step 4.** Solve the resulting equation:

$x-1 = 0$
$x = 1/2$

Because the first term and coefficient were perfect squares and the middle term was negative, **if this trinomial is factorable**, we have a perfect square trinomial that is the difference of two terms.

**Example:** $12x^3+5x^2 = 2x$

**Step 1.** Rewrite in standard form:
$12x^3+5x^2-2x = 0$.

**Step 2.** Factor completely. ***Note:*** *first, factor out the greatest common factor (GCF)* $x(12x^2+5x-2)$. ***Then*** *factor the trinomial:*
$x(4x-1)(3x+2)=0$

**Step 3.** Set **each** factor containing a variable equal to 0: $x = 0$, $4x-1 = 0$, and $3x+2 = 0$.

**Step 4.** Solve the resulting equations:

| | | |
|---|---|---|
| $x = 0$ | $(4x-1) = 0$ | $(3x+2) = 0$ |
| | $4x = 1$ | $3x=-2$ |
| | $x = 1/4$ | $x = - 2/3$ |

**Don't forget your GCF! THIS IS ALWAYS THE FIRST STEP WHEN FACTORING.** If is contains a variable (or is a variable as in this case), be sure to set it equal to zero too.

**Example:** $x(4x-11) = 3$

**Step 1.** Rewrite in standard form: $4x^2-11x-3 = 0$. ***NOTE:*** *Before we rewrite this polynomial in standard form, we must* ***simplify the equation.*** *Remove the parenthesis by distributing the x through the factor (4x-11). Then, rewrite the resulting equation to be equal to 0.*

**Step 2.** Factor completely $(4x+1)(x-3)$. ***NOTE:*** *This is not a "perfect square trinomial" the first term cannot be factored as (2x)(2x).*

**Step 3.** Set each factor containing a variable equal to 0:
$4x+1 = 0$ and $x-3 = 0$

**Step 4.** Solve the resulting equations:

| | |
|---|---|
| $4x+1 = 0$ | $x-3 = 0$ |
| $4x = -1$ | $x = 3$ |
| $x = -1/4$ | |

**The Fundamental Theorem of Algebra**

Look at the examples given. Compare the number of solutions with the degree of the polynomial. The number of solutions to any polynomial equation is ALWAYS less than or equal to the degree of the polynomial. This fact is known as the fundamental theorem of algebra.

# Additional Key Concepts: Factoring Polynomials

**The Greatest Common Factor**

**Factoring** is the process of writing an expression as a product.

**The GCF of a list of common variables raised to powers** is the variable raised to the smallest exponent in the list.

**The GCF of a list of** terms is the product of all common factors.

**Factoring by Grouping:**

1. Group the terms into two groups of two terms.
2. Factor out the GCF from each group.
3. If there is a common binomial, factor it out.
4. If not, rearrange the terms and try steps 1-3 again.

**Factoring Trinomials in the Form $x^2+bx+c$**

$x^2+bx+c = (x+?)(x+?)$, where the numbers indicated by the "?" sum to "b" and the product of the numbers indicated by the "?" is "c"

**Factoring Trinomials in the Form $ax^2+bx+c$**

To factor $ax^2+bx+c$, try various combinations of factors of $ax^2$ and c until the middle term of *bx* is obtained when checking (the product of the outside term and the product of the inside terms sum to equal the middle term).

**Factoring Trinomials in the Form $ax^2+bx+c$ by Grouping**

1. Find two numbers whose product is a*c and whose sum is b
2. Rewrite bhx, using the factors found in step 1.
3. Factor by grouping.

**Factoring Perfect Square Trinomials** (Trinomials that are the square of some binomial)

- $a^2+2ab+b^2 = (a+b)^2$
- $a^2-2ab+b^2 = (a-b)^2$

**Difference of Two Squares**

- $a^2-b^2 = (a+b)(a-b)$

# Online Resources

**Quadratic Equations**
http://www.mathpower.com/tut99.htm

**Quadratic Equations: Solutions by Factoring**
http://www.sosmath.com/algebra/quadraticeq/sobyfactor/sobyfactor.html

http://www.mathpower.com/tut105.htm

http://www.mathpower.com/tut110.htm