# Algebra Connections

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# Rational Expressions


A **rational number** is a number that can be written as a quotient of integers. A **rational expression** is an expression that can be written as a quotient, or the form: P/Q, where P and Q are polynomials and Q does not equal 0.

Examples of rational expressions include:

$$\frac{x^2-1}{x+8},\ or\ \frac{3a^2+5a-3}{a-9}$$

$$or\ \frac{5x^2-3x+2}{3x+7}$$

$$or\ \frac{4x^2-6x-5}{1}$$

The last example above could be expressed as $4x^2-6x-5$, a polynomial.

Like all real numbers, dividing by the integer 1 maintains the same identity or value. Dividing by 1 turns our polynomial into a rational expression.

Rational expressions are also referred to as **rational functions,** because the identity or value of the equation is a result (or function) of the number(s) we substitute for the variable(s).

The set of values of real numbers that can be substituted into a function and result in a real number is called the **domain.**

Values that cause a rational expression to be undefined (the denominator becomes 0) are not part of the domain. Recall that we never divide by zero.

## Undefined Rational Expressions

With rational expressions (rational functions), we need to watch out for values that cause our denominator to be 0, an undefined value. When looking for the domain of a rational function (the set of values that result in a real number), find the values that cannot be used, values that make the denominator 0.

**Example 1:** For what values of $x$ is the rational expression undefined?

$$\frac{y-3}{2y+7}$$

Therefore we are solving:

$$2y+7=0$$
$$2y+7-7=0-7$$
$$2y=-7$$
$$(2y)/2=-7/2$$
$$y=-7/2$$

For the value –7/2, this rational expression is undefined – in other words, the domain of this rational expression is every value of y EXCEPT –7/2. The denominator of a fraction can never be equal to 0.

Any number divided by zero is undefined – it is not a real number. Any value(s) of $x$ which causes the denominator to equal 0 cannot be part of the domain of a rational expression.

**Example 2**. For what values of $x$ are the rational expression undefined?

$$\frac{x^2+2}{x^2-3x+2}$$

We are solving: $x^2-3x+2=0$

The first thing we must do is factor the denominator:

$$x^2-3x+2=(x-2)(x-1)$$

Now we set each of those factors equal to 0 and solve:

| | |
|---|---|
| $x-2=0$ | $x-1=0$ |
| $x-2+2=0+2$ | $x-1+1=0+1$ |
| $x=2$ | $x=1$ |

We see that when x = 2 or 1, we have a denominator of 0. This rational expression is undefined when x = 2 and x = 1.

## Simplifying a Rational Expression

Recall that we defined a fraction as being in "simplest" form or "lowest terms" when the numerator and denominator have no common factors other than 1 or –1. We also need to simplify rational expressions.

A rational expression, P/Q ($Q\neq0$) is in its simplest form or lowest term if the greatest common factor of its numerator and denominator is 1.

Rational expressions represent a real number for each value of the variable that does not make the denominator zero. All properties of real numbers apply to rational expressions.

We use the fundamental property of rational expressions to write a rational expression in lowest terms.

### Fundamental Principle of Rational Expressions

The Fundamental Principle of Rational Expressions is that if P/Q is a rational expression and *R* is a nonzero polynomial, then:

$$\frac{PR}{QR} = \frac{p}{Q}$$

Simplifying a rational expression is similar to simplifying a ratio or fraction. Factor the numerator and the denominator. Look for common factors, and eliminate them using the **Fundamental Principal of Rational Expressions** (common factors cancel out).

Just like with fractions, when we rewrite a rational expression without the common factors in the numerator and denominator, it is in simplest form.

**Example:** Write each expression in lowest term

$$\frac{30}{72} = \frac{2\times 3\times 5}{2\times 2\times 2\times 3\times 3}$$
$$= \frac{5(2\times 3)}{2\times 2\times 3(2\times 3)}$$
$$= \frac{5}{2\times 2\times 3} = \frac{5}{12}$$

Note that this is an application of the fundamental property. Likewise:

$$\frac{14a^2}{2a^3} = \frac{2\times 7\times a\times a}{2\times a\times a\times a}$$
$$= \frac{7(2\times a\times a)}{a(2\times a\times a)} = \frac{7}{a}$$

Note how we used the **Quotient Rule of Exponents** when numerators and denominators have the same base.

$$\frac{a^m}{a^n} = a^{m-n}$$

**Example:** Simplify $\frac{18a^4}{9a^5}$

$$\frac{18a^4}{9a^5} = \frac{18}{9}\times\frac{a^4}{a^5} = 2a^{4-5} = 2a^{-1} = \frac{2}{a}$$

### To Simplify A Rational Expression:

1. Completely factor the numerator and denominator
2. Apply the fundamental principle of rational expressions to divide out common factors. **Note:** *This is what the Greatest Common Factor does.*

**Example:** Simplify $\frac{10x-10}{x^3-x^2}$

$$= \frac{10(x-1)}{x^2(x-1)} = \frac{10}{x^2}$$

**Example:** Simplify $\frac{x^2-16}{x^2-8x+16}$

$$= \frac{(x+4)(x-4)}{(x-4)(x-4)} = \frac{x+4}{x-4}$$

### Points to Remember

When simplifying rational expressions, **the fundamental principle applies to common factors, NOT common terms.**

$$\frac{x(x+2)}{x\times x} = \frac{x+2}{x}$$

Note that, on the right, we cannot factor out another x from the numerator and denominator. They are common terms, **NOT** factors.

Also remember that we can always factor out -1. That will be necessary when we have factors that only differ by signs. Factoring out a -1 lets us rewrite an expression using opposite signs.

**Example:** Simplify $\frac{25-x^2}{x^2-10x+25}$

$$= \frac{(5-x)(5+x)}{(x-5)(x-5)}$$
$$= \frac{-1(x-5)(5+x)}{(x-5)(x-5)} = -\frac{5+x}{x-5}$$

**Example:** Simplify $\frac{x^2+7x+12}{x^2-6x-27}$

$$= \frac{(x+4)(x+3)}{(x+3)(x-9)} = \frac{x+4}{x-9}$$

### Rational Expression = Fractions

Rational expressions are quotients of two algebraic expressions. They can be manipulated like fractions. The following properties of fractions apply to rational expressions:

- For any real numbers a and b, where $b \neq 0$: -a/b = a/-b = -(a/b)
- For any real numbers a and b, where $b \neq 0$: -a/-b = a/b
- For any real numbers a, b and k, where $b \neq 0$ and $k \neq 0$: (a·k)/(b·k) = a/b
- For any real numbers a and b, where $a \neq b$: (a-b)/(b-a) = -1

## Online Resources: Rational Expressions

**Interactive Algebra Reviews: Rational Expressions**

http://people.hofstra.edu/faculty/Stefan_Waner/RealWorld/tut_alg_review/framesA_4.html

http://www.mathnotes.com/Combined/aw_comboch8.html

**University Of Utah Online Rational Expression Review**

http://www.math.utah.edu/online/1010/rational/

**Short online quizzes for rational expressions**

http://library.thinkquest.org/11771/english/hi/math/tests/alg/5.html

http://www.glencoe.com/sec/math/studytools/cgi-bin/msgQuiz.php4?isbn=0-07-825083-8&chapter=11&lesson=1&headerFile=4&state=tx

**Rational Expressions: Simplifying**

http://www.purplemath.com/modules/rtnldefs.htm

http://www.jamesbrennan.org/algebra/rational/simplifying_rational_expressions.htm

**Rational Expressions**

http://www.marlboro.edu/academics/study/mathematics/courses/f04emls/rational.html

http://www.edteach.com/algebra/rational/simplifying_rational_expressions.htm

http://www.tcc.fl.edu/dept/acsu/mc/docs/pdf/algebra/smrtexpr.pdf

http://www.csun.edu/~hfmth006/Math094/094b/mod9/mod9ch1.pdf

http://www.bestquest.com/printmaterial/AS_15_2_BlackLineMasters.pdf