Algebra Connections

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Adding & Subtracting Rational Expressions

In many ways, working with rational expressions is like working with fractions. Recall that:

$$\frac{a}{b}+\frac{c}{b}=\frac{a+c}{b}, b \text{ does not equal } 0$$

$$\frac{a}{b}-\frac{c}{b}=\frac{a-c}{b}, b \text{ does not equal } 0$$

As long as we have common denominators, adding and subtracting rational expressions is fairly straightforward – simply rewrite the addition or subtraction on the common denominator.

$$\frac{P}{R}+\frac{Q}{R}=\frac{P+Q}{R}$$

$$\frac{P}{R}-\frac{Q}{R}=\frac{P-Q}{R}$$

R does not equal 0

To add or subtract rational expressions with different denominators, we need to find the Lowest Common Denominator for the polynomials that represent the denominators or our rational expressions.

To find the LCD for polynomials:

1. Factor each denominator completely. Use exponent notation for repeated factors.
2. Write the product of all the different factors that appear in the denominators.
3. On each factor, use the highest power that appears on that factor in any or the denominators.

Example: LCD of 20, 50

1. Factor completely: $20=2^2*5$, $50=2*5^2$
2. Product of all different factors: 2 and 5
3. Use highest power: $2^2*5^2=100$

Example: LCD x^3yz^2, x^5y^2z, xyz^5

1. Factor completely, each expression already is a factor: x^3yz^2, x^5y^2z, xyz^5.
2. Product of all the different factors (x, y, z)
3. Use highest power: $x^5y^2z^5$

Example: LCD a^2+5a+6, a^2+4a+4

1. Factor completely: $(a+2)(a+3)$ and $(a+2)^2$
2. Product of all different factors: (a+2), (a+3), and (a+2)
3. Use highest power: $(a+3)(a+2)^2$

Adding and Subtracting Rational Expressions with Different Denominators

When we add or subtract rational expressions with different denominators, we must convert them to contain identical denominators. The easiest way to do this is to use the LCD that we have just reviewed. Once the LCD is determined, rewrite each rational expression as an equivalent rational expression with the LCD. The process is just like working with fractions.

Example: $\frac{3}{20}+\frac{7}{12}$

1. Factor completely: $20=2^2*5$, $50=2*5^2$
2. Product of all different factors: 2 and 5
3. Use highest power: $2^2*5^2=100$

$$=\frac{3}{20}\left(\frac{3}{3}\right)+\frac{7}{12}\left(\frac{5}{5}\right)=\frac{9}{60}+\frac{35}{60}$$

$$=\frac{44}{60}=\frac{4*11}{4*15}=\frac{11}{15}$$

Example: $\frac{1}{6}-\frac{4}{15}$

1. Factor completely: 6=2*3, 15=3*5
2. Product of all different factors: 2, 3, and 5
3. Use highest power: 2*3*5=30

$$\frac{1}{6}\left(\frac{5}{5}\right)-\frac{4}{15}\left(\frac{2}{2}\right)=\frac{5}{30}-\frac{8}{30}$$
$$=\frac{-3}{30}=\frac{-1*3}{10*3}=-\frac{1}{10}$$

Rational expressions are added and subtracted just like rational numbers (quotients of rational number or fractions). When necessary, rewrite rational expressions with a common denominator.

Follow these steps to add or subtract rational expressions when the denominators differ.

1. Find the Lowest Common Denominator (LCD)
2. Rewrite each rational expression as an equivalent expression whose denominator is the LCD.
3. Add or subtract numerators and place the sum or difference over the common denominator.
4. Write the result in lowest terms.

Example: $\frac{5}{2x}+\frac{2}{3}$

$$=\frac{5}{2x}\left(\frac{3}{3}\right)+\frac{2}{3}\left(\frac{2x}{2x}\right)$$
$$=\frac{15+4x}{6x}$$

Example: $\frac{1}{x^2-9}+\frac{2}{x^2+3x}$

$$=\frac{1}{(x-3)(x+3)}+\frac{2}{x(x+3)}$$
$$=\frac{1}{(x-3)(x+3)}\left(\frac{x}{x}\right)+\frac{2}{x(x+3)}\left(\frac{x-3}{x-3}\right)$$
$$=\frac{x+2x-6}{x(x-3)(x+3)}=\frac{3x-6}{x(x-3)(x+3)}$$

Example: $\frac{4}{5-a}-\frac{2}{a-5}$

Note: because (5-a) and (a-5) differ only in signs, we can obtain identical denominators by multiplying only the first expression by -1 in BOTH the numerator and denominator (maintain and equivalent expression).

$$=\frac{4}{5-4}\left(\frac{-1}{-1}\right)-\frac{2}{a-5}$$
$$=\frac{-4}{a-5}-\frac{2}{a-5}$$
$$=\frac{-6}{a-5}=-\frac{6}{a-5}$$

Example: $\frac{x+1}{x^2+2x}+\frac{2x+1}{6x+12}-\frac{1}{6}$

Note: The LCD for these three denominators is 6x(x+2)

$$=\frac{x+1}{x(x+2)}+\frac{2x+1}{6(x+2)}-\frac{1}{6}$$
$$=\frac{x+1}{x(x+2)}\left(\frac{6}{6}\right)+\frac{2x+1}{6(x+2)}\left(\frac{x}{x}\right)-\frac{1}{6}\left(\frac{x(x+2)}{x(x+2)}\right)$$
$$=\frac{6x+6}{6x(x+2)}+\frac{2x^2+x}{6x(x+2)}-\frac{x^2+2x}{6x(x+2)}$$
$$=\frac{6x+6+2x^2+x-x^2-2x}{6x(x+2)}$$
$$=\frac{x^2+5x+6}{6x(x+2)}=\frac{(x+3)(x+2)}{6x(x+2)}$$
$$=\frac{x+3}{6x}$$

Online Resources

Finding the Lowest Common Denominator

http://www.csun.edu/~ayk38384/Math093-Rational%20Expression.htm

http://library.thinkquest.org/20991/textonly/alg/frac.html

Adding and Subtracting Rational Expressions

http://www.mathsteacher.com.au/year10/ch14_rational/05_addition_and_subtraction/addsub.htm

http://a-s.clayton.edu/garrison/Math%200099/sect44.htm

http://faculty.ed.umuc.edu/~swalsh/Math%20Articles/RationalE.html

http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut33_addrat.htm

http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut33_addrat.htm

http://www.algebra-online.com/adding-substracting-rational-expressions-like-denominators-1.htm

http://tutorial.math.lamar.edu/AllBrowsers/1314/RationalExpressions.asp

http://www.sparknotes.com/math/algebra2/rationalexpressions/section2.rhtml

Purple Math's Tutorial: Adding and Subtracting Rational Expressions (2 parts)

http://www.purplemath.com/modules/rtnladd.htm

http://www.purplemath.com/modules/rtnladd2.htm