Algebra Connections

Mr. Breitsprecher's Edition | **April 4, 2005** | **Web: www.clubtnt.org/my_algebra**

Multiplying & Dividing Rational Expressions

Multiplying and dividing rational expressions is just like multiplying and dividing fractions – they just look more complicated. Recall that:

$$\frac{a}{b}\times\frac{c}{d}=\frac{ac}{bd} \quad \textit{where } bd \neq 0$$

$$\frac{a}{b}\div\frac{c}{d}=\frac{a}{b}\times\frac{d}{c}=\frac{ad}{bc} \quad \textit{where } bd$$

When working with fractions, we can perform the arithmetic and then look at our results to see if it is in "***lowest terms***." Many of us, with practice, can intuitively see how to simplify fractions because we recognize when numbers contain common factors. We are able to "***see***" the common factors without factoring the numerators and denominators.

Also notice that dividing fractions is simply multiplying by the inverse. Once we are comfortable multiplying fractions, we will quickly learn to divide them if we are able to remember to rewrite the division as multiplication. Instead of dividing, multiply by the inverse.

Multiplying Rational Expressions

Multiplying rational expressions works exactly like fractions, but the expressions are more complicated. Not only are we working with numerators and denominators, each are a polynomial. We need a way to keep the polynomials in the numerators and denominators manageable.

Few of us will intuitively see all common factors in rational expressions. The key will be to completely factor the numerators and denominators **FIRST**, then cancel out common factors (which is applying the **Fundamental Principal of Rational Expressions**). The factors that are left represent our answer and this answer is expressed in its simplest form.

Do you see that we **DO NOT ACTUALLY PERFORM ANY MULTIPLICATION WHEN WE MULTIPLY RATIONAL EXPRESSIONS?**

We factor, which means write rational expression as a product, and then cancel out all common factors that appear in **BOTH** the numerator and denominator (Fundamental Principal of Rational Expressions). What is left after factoring and canceling out common factors between the numerator and denominator **IS OUR PRODUCT!**

Steps in Multiplying Rational Expressions

Stated formally, as in a typical Algebra Textbook, the process for multiplying rational expressions that we have been discussing can be written as:

1. Factor numerators and denominators.
2. Multiply Numerators and multiply denominators.
3. Write the product in lowest terms (apply Fundamental Principal of Rational Expression, PR/QR = P/Q, where R is a nonzero polynomial)

Note that we can apply these steps **BY COMBINING 1 & 2** by factoring the numerators and nominators and rewriting each of these sets of factors as 1 rational expression that contains ALL factors from each numerator and denominator.

This is how ***Algebra Connections*** will present its examples. We will completely factor each rational expression, writing the factors as one rational expression, keeping factors of the numerators on top and factors of the denominators on the bottom.

Example 1: $\frac{9x}{5y} \times \frac{10y}{3xy}$

$= \frac{(3 \times 3x) \times (2 \times 5y)}{5y \times 3xy}$

$= \frac{6}{y}$

Example 2: $\frac{-8xy^4}{3z^3} \times \frac{15z}{2x^5y^3}$

$= \frac{(-2 \times 2 \times 2xy^4) \times (3 \times 5z)}{(3z^3) \times 2x^5y^3}$

Example 3: $\frac{x^2+7x+12}{2x+6} \times \frac{x}{x^2-16}$

$= \frac{[(x+3) \times (\times+4)] \times x}{[2(x+3)] \times (x-4)(x+4)}$

$= \frac{x}{2(x-4)}$

$= \frac{x}{2x-8}$

Dividing Rational Expressions

Just like fractions – once we are comfortable with the process of multiplying rational expressions, we are ready to divide if we remember to rewrite our division as multiplication.

Instead of dividing, multiply by the reciprocal. As we have just seen, that means to factor completely, cancel out common terms that appear in **BOTH** the numerator and denominator, and write down the remaining factors.

Note that we can write division of rational expressions with the "÷" sign or with a fraction bar.

Example 4: $\frac{\frac{5}{3x}}{\frac{5}{6x}} = \frac{5}{3x} \div \frac{5}{6x}$

Either way, we **invert the divisor and multiply.**

Example 4: $\frac{5}{3x} \div \frac{5}{6x}$

$= \frac{5}{3x} \times \frac{6x}{5} = \frac{5 \times (2 \times 3x)}{3x \times 5} = \frac{2}{1} = 2$

Example 5: $\frac{x^7}{2} \div (2x^{2)}$

$= \frac{x^7}{2} \times \frac{1}{2x^2} = \frac{x^7 \times 1}{2 \times 2x^2} = \frac{x^5}{4}$

Example 6: $\frac{2x+1}{x+5} \div \frac{6x^2-x-2}{x^2-25}$

$= \frac{2x+1}{x+5} \times \frac{x_2-25}{6x^2-x-2}$

$= \frac{(2x+1) \times [(x+5)(x-5)]}{(x+5) \times [(2x+1)(3x-2)}$

$= \frac{x-5}{3x-2}$

Indicated Division with Fraction Bar

Many beginning algebra books, such as **K. Elayn Martin-Gay's *Introductory Algebra***, do not present division of rational expressions with fraction bars – they introduce this concept as "**complex fractions.**"

Remember, fractions always indicate division. **The denominator is the divisor.** The result of division is called the **quotient.** Regardless of how we write division (fraction bar or "÷"), we need to invert the divisor (its reciprocal) and rewrite the problem as multiplication.

Example 7: $\frac{\frac{a+b}{3}}{\frac{1}{6}}$

$= \frac{a+b}{3} \times \frac{6}{1} = \frac{(a+b) \times (2 \times 3)}{3 \times 1}$

$= (a+b)2 = 2a+2b$

Example 8: $\frac{\frac{x^2-1}{2}}{\frac{x-1}{3}}$

$= \frac{x^2-1}{2} \times \frac{3}{x-1} = \frac{[(x-1)(x+1)] \times 3}{2 \times (x-1)}$

Example 9: $\frac{\frac{a^2+5}{3}}{2}$

$= \frac{a^2+5}{3} \times \frac{1}{2} = \frac{a^2+5}{6}$

Online Resources

Multiplying Rational Expressions

http://www.purplemath.com/modules/rtnlmult.htm

http://www.algebra-online.com/multiplying-rational-expressions-1.htm

Dividing Rational Expressions

http://www.purplemath.com/modules/rtnlmult2.htm

http://www.algebra-online.com/dividing-rational-expressions-1.htm

Multiplying & Dividing Rational Expressions

http://www.sci.wsu.edu/~kentler/Fall97_101/nojs/Chapter7/section1.html

http://www.wtamu.edu/academic/anns/mps/math/mathlab/col_algebra/col_alg_tut9_mulrat.htm

http://www.wtamu.edu/academic/anns/mps/math/mathlab/int_algebra/int_alg_tut32_multrat.htm

http://faculty.ed.umuc.edu/~swalsh/Math%20Articles/RationalE.html

Multiplying, Dividing, Adding, and Subtracting Rational Expressions

http://tutorial.math.lamar.edu/AllBrowsers/1314/RationalExpressions.asp

Factoring a Polynomial:

3. Are there any common factors? If so, factor them out.
4. How many terms are in the polynomial:
   a. **Two terms:** Is it the difference of 2 squares? $a^2-b^2 = (a-b)(a+b)$
   a. **Three terms:** Try one of the following patterns
      i. $a^2+2ab+b^2 = (a+b)^2$
      ii. $a^2-2ab+b^2 = (a-b)^2$
      iii. Otherwise, try to use another method.
   b. **Four terms**: Try factoring by grouping.
1. See if any factors can be factored further. Watch for difference of 2 squares!

Quick Review: Two Important Exponent Rules

- $a^m * a^n = a^m +^n$
- $a^m / a^n = a^{m-n}$