Algebra Connections

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Solving Linear Equations in 1 Variable

In order to understand linear equations in 1 variable, recall that a **variable** is a number that is not identified. It is often represented by "x" or "y," any letter can be used. A **linear expression** is a mathematical statement that performs functions of addition, subtraction, multiplication, and division, but has no exponents (or powers) and no variables that multiply or divide each other. Some examples of linear expressions include:

- $x + 4$
- $2x = 4$
- $2x + y$

The following examples **ARE NOT** linear expressions:

- x^2
- $2xy + 4$
- $2x / 4y$

A **linear equation** is a mathematical statement that has an equal sign and linear expressions. Examples include:

- $2x + 4 = 10$ (linear equation in 1 variable)
- $3x - 4 = -10$ (linear equation in 1 variable)
- $4x - 4y = 8$ (linear equation in 2 variables)

Solving linear equations involves applying logical steps to simplify the expressions while still maintaining the equality and original identity or solution. Each step creates an equivalent equation, not a new one. Like most procedures, this will be easiest to learn if we establish a sequence of steps and practice using them until we are comfortable.

Most of us would agree that math problems are easier to work with if we **ELIMINATE OR CLEAR ALL FRACTIONS** first. To **SOLVE LINEAR EQUATIONS IN ONE VARIABLE**, practice the following 6-step process:

1. Multiply both sides of the equation to **CLEAR FRACTIONS** if they occur – multiply both sides of the equation by the lowest common denominator.
2. Use the distributive property to **REMOVE PARENTHESES** if they occur. This will often create "like terms."
3. Simplify both sides of the equation, **COMBINE LIKE TERMS.**
4. Get all terms containing variables, all **VARIABLE TERMS ON ONE SIDE OF THE EQUATION** and all numbers on the other side by using the addition property of equality.
5. **GET THE VARIABLE ALONE** by using the multiplication property of equality to remove the coefficients of any term with a variable.
6. **CHECK YOUR SOLUTION** by substituting it into the original equation.

Let's look at how the examples of linear equations in one variable, identified earlier, would be solved by applying our set of logical procedures.

We will not look at our 3rd example of a linear equation because it is a linear equation in 2 variables.

Example 1: $2x + 4 = 10$

1. Note that there are no fractions, so we will go to step 2.
2. There are no parentheses to remove, so we will go to step 3.
3. There are no like terms to combine, so we will go to the next step, 4.
4. Isolate "x" to one side of the equation with the addition property of equality. Add the opposite, a –4, on both sides of the equation to move that term to the other side: $2x + 4 - 4 = 10 - 4$. This simplifies to $2x = 6$. We now have only the term with the variable on the left, but it has a coefficient.

Give Me Life, Liberty, And The Pursuit Of Algebraic Properties Of Equality

Whether solving simple algebraic expressions, linear equations, or a host of other types of problems, applying the basic properties of real numbers is the key to success. Using these tools, we can work towards isolating variables and solving for the unknown. Here is a quick review of some of the algebraic properties that are used to simplify linear equations.

Transitive Property of Equality. Property indicates the logic behind equalities and how it can create inferences about equalities. If a = b and b = c, then a = c.

Commutative Property of Addition. Numbers can be added in any. a + b = b + a

Associative Property of Addition. When adding three numbers, it doesn't matter if the first two or the last two numbers are added first. (a + b) + c = a + (b + c)

Commutative Property of Multiplication. Numbers can be multiplied in any order. For example: a * b = b * a

Associative Property of Multiplication. When multiplying three numbers, it doesn't matter if the first two or the last two numbers are multiplied first. (a * b) * c = a * (b * c)

Distributive Property. Property indicating a special way in which multiplication is applied to addition of two (or more) numbers. For example: a(b + c) = a*b + a*c

Addition Property of Equality. If you add the same number to each side of an equation, the two sides remain equal. If a = b, then a + c = b + c. If a = b, then a + c = b + c.

Multiplication Property of Equality. If each side of an equation is multiplied by the same number, then the two sides remain equal. 3 = 2 + 1, then 3 x 4 = (2+1) x 4. And if a = b, then ac = bc. If a = b, then a * c = b * c.

Online Resources

Earlier this year, Algebra Connections published some Internet resources for some of these skills. Here are more interactive tutorials and reviews. Sorry for the long URL's. If you have any problems, just enter the domain name (http://domain_name.org) WITHOUT the directories (delete everything after the first forward slash /). Then follow the links on that page to the sites listed, using the directories as a guide. Remember, there are never spaces in a URL, please use the underscore where indicated.

Addition Property of Equality
http://www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut12_addeq.htm

Multiplication Property of Equality
http://www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut13_multeq.htm

Solving Linear Equations (Algebra Lab – great site!).
http://www.algebralab.org/practice/practice.aspx?file=Algebra2_1-3.xml

Solving Linear Equations
http://www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut14_solve.htm

Solving Linear Equations
http://education.yahoo.com/college/student_life/math_homework/problem_list?id=minialg1gt_1_1

5. Isolate the variable (remove the coefficient) by using the multiplication property of equality. Multiplying both sides by 1/2. Now we have: 2x*(1/2) = 6*(1/2). This will simplify to x=3.
6. Check your answer in the original equation, 2x+4 = 10, by substituting our proposed solution (x=3) for the variable. (2*3)+4 = 10. This simplifies to: 6+4 = 10. This is a true statement; we have a solution to this linear equation in one variable.

Example 2: 3x - 4 = -10

1. Note that there are no fractions, so we will go to step 2.
2. There are no parentheses to remove, so we will go to step 3.
3. There are no like terms to combine, so we will go to the next step, 4,
4. Isolate "x" to one side of the equation with the addition property of equality. Add the opposite, a 4, on both sides of equation to move that term to the other side: 3x - 4 + 4 = -10 + 4. This simplifies to 2x = -6. We now have only the term with the variable on the left, but it has a coefficient.
5. Isolate the variable (remove the coefficient) by using the multiplication property of equality. Multiplying both sides both sides by 1/3. Now we have: 3x *1/3 = -6*1/3. This will simplify to x = -2
6. Check your work in the original equation, 3x-4 =-10, by substituting our proposed solution (x=-2) for the variable. (3*-2)-4 = -10; -6-4 = -10. This is a true statement and we have a solution to this linear equation in one variable.