# Algebra Connections

Mr. Breitsprecher's Edition | February 14, 2005 | Web: www.clubtnt.org/my_algebra


# Solving Linear Inequalities: $< > \leq \geq$

The following are examples of inequalities:

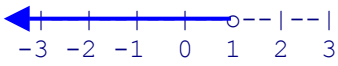
- $a < b$ a is less than b
- $a \leq b$ a is less than or equal to b
- $a > b$ a is greater than b
- $a \geq b$ a is greater than or equal to b

## Graphing Inequalities

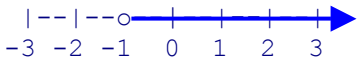
When we draw equalities on a number line, we merely have to place a dot or point on the location indicated. Graphing inequalities is not hard, but we need to indicate the entire set of numbers that are part of that inequality – it is not just one point, there are infinite points.

We will do this by using a "o" for the point that is **NOT** included in our graph of an inequality. We will then draw an arrow from that point to show which way the set of numbers that represents that inequality extends.

**Example:** $x < 1$



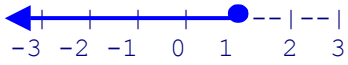

**Example:** $x > -1$



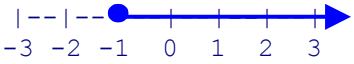

Remember, **EVERY** inequality represents a **SET** of numbers, to the left or right of where we start.

When we graph inequalities that include "equal to" such as "greater than or equal to" or "less than or equal to," we need to **INCLUDE** the starting point in our graph. We do this by using "●" and drawing our arrow to represent the set from that point.

**Example:** $x \leq 1$




**Example:** $x \geq -1$




In order to understand linear inequalities, recall that a **variable** is a number that is not identified. It is often represented by "x" or "y," any letter can be used.

A **linear expression** is a mathematical statement that performs functions of addition, subtraction, multiplication, and division, but has no exponents (or powers) and no variables that multiply or divide each other.

A **linear inequality** is a mathematical statement that has one of the inequality signs above and linear expressions.

The examples of linear equalities discussed so far are fairly straight-forward. The graphs will become intuitively obvious with practice. When inequalities are not so obvious, we can use a similar process to that which is used to solve linear equations. The properties of equalities are similar to the properties of inequality.

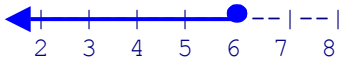
## Addition Property for Inequalities

If $a < b$, then $a + c < b + c$

In other words, adding or subtracting the same expression to both sides of an inequality does not change the inequality.

**Example:** $x - 5 \leq 1$

- $x - 5 \leq 1$
- $x - 5 + 5 \leq 1 + 5$
- $x \leq 6$




Recall that the product of 2 real numbers with the same sign is ALWAYS positive. Also recall that the product of 2 numbers with different signs is **ALWAYS** a negative number.

## Multiplication Properties for Inequalities

- When multiplying by a positive value, If $a < b$ **AND** c is positive, then $ac < bc$
- When multiplying by a negative value, If $a < b$ **AND** c is negative, then $ac > bc$.

In other words, multiplying the same **POSITIVE** number to both sides of an inequality **DOES NOT CHANGE** the inequality. Multiplying the same **NEGATIVE** number to both sides of an inequality **REVERSES** the sign of the inequality.

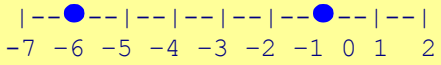
# Solving Linear Inequalities is the Same as Solving Linear Equalities, EXCEPT…

Yes, if you have mastered keeping track of positive and negative signs when performing multiplication (we have defined division in terms of multiplication), solving linear equations is the same **WITH ONE VERY IMPORTANT EXCEPTION.**

Let's look at an example and see if we can make this clear. Look at this statement, 6 > 3. Certainly, we would all agree that 5 is greater than 3.

Now we will multiply both sides by a convenient number, let's use –1; however, any negative number would do. Is (-1)(6) still greater than (3)(-1)? **NO!!!!!**

**-6 is not greater than –1!**
Notice that it is left than –1 on the number line:




Notice that multiplying by a negative number changes the sign

- A positive number multiplied by a negative number results in a negative number.
- A negative number multiplied by a negative number is a positive number.

Multiplying **BOTH** the left and right side of an equation by a negative number is never an issue (equations contain an "=" sign). **IT IS ALWAYS AN ISSUE** when we multiply **BOTH** the left and right side of an **INEQUALITY** by a negative number. **WE HAVE TO "FLIP," (or change the direction ) OF THE SIGN!** This is also true if you divide by a negative.

## Online Resources

**Note:** These URL's can be long and frustrating to enter. You should be able to go to the domain name (domain_name.com) and find links to that will take you to these pages.

**Algebra: Solving Linear Inequalities**
http://en.wikibooks.org/wiki/Algebra:Solving_linear_inequalities

**S.O.S. Math: Inqualities**
http://www.sosmath.com/algebra/inequalities/ineq01/ineq01.html

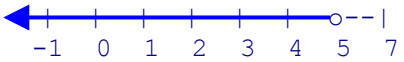
**Math.com: Inequalities**
http://www.math.com/school/subject2/lessons/S2U3L4DP.html

**Mathematics Help Central: Solving Linear Inequalities**
http://www.mathematicshelpcentral.com/lecture_notes/intermediate_college_algebra/solving_linear_inequalities.htm

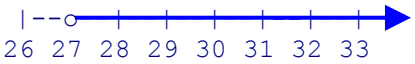
**ThinkQuest. Solving Linear Equations**
http://library.thinkquest.org/C0110248/algebra/ielinearsolving.htm

**Multiplication Properties for Inequalities** (continued)

**Example:** 5x < -10
- (1/5) * 5x < -10 * (1/5)
- x < (10/5)
- x < 5

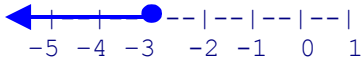



**Example:** (x / 3) > 9
- 3 * (x / 3) > 9 * 3
- [(3x) / 3] > 27
- x > 27




In the next examples, we will multiply BOTH sides of the equation by a negative number. T IS ALWAYS MEANS we need to change the direction of the inequality sign.

**Example:** (-3x) ≥ 9 **Note:** The coefficient of the first term is (-3). Therefore, we will multiply **BOTH SIDES** by (-1/3). The inequality needs to be reversed to make the statement true.
- (-1/3) * (-3x) ≤ 9 * (-1/3)
- x ≤ -9/3
- x ≤ -3




If you are unsure about this, please choose any number from our solution and check it in our original equation. You will get a "true" statement.

**Example:** (-x/3) > 7. **Note:** The coefficient of the first term is (-1/3). Therefore, we will multiply **BOTH SIDES** by (-3). The inequality needs to be reversed to make the statement true.
- -3 * (-x / 3) < 7 * -3
- x > -27